

**A STANDARD COMPUTABLE GENERAL
EQUILIBRIUM (CGE) MODEL IN GAMS**

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Abstract

Computable general equilibrium (CGE) models are used widely in policy analysis. The purpose of this paper is to contribute to and facilitate the use of CGE models. The paper includes a detailed presentation of a “standard” CGE model (an equation-by-equation description) and its required database. It incorporates features developed in recent years in research projects conducted at IFPRI. These features, which are of particular importance in developing countries, include household consumption of non-marketed (or “home”) commodities, explicit treatment of transaction costs for commodities that enter the market sphere, and a separation between producing activities and commodities that permits any activity to produce multiple commodities and any commodity to be produced by multiple activities. The paper discusses the implementation of the model in GAMS (the General Algebraic Modeling System) and is accompanied by a self-extracting zip file, which includes the GAMS files for the model, sample databases, simulations, solution reports, and a SAM aggregation program. Although the paper provides a standardized framework for analysis, the analyst is not forced to make “one-size-fits-all” assumptions. The GAMS code is written in a manner that gives the analyst considerable flexibility in model specification.

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1. INTRODUCTION

In recent years, improvements in model structures, databases, and computer technology have improved the payoffs and reduced the costs of policy analysis based on Computable General Equilibrium (CGE) models, paving the way for their widespread use by analysts throughout the world.

The purpose of this paper is to contribute to and facilitate the use of model structures adapted to the characteristics of developing countries. The paper includes a detailed presentation of a “standard” CGE model and its required database. The model follows the neoclassical-structuralist modeling tradition that is presented in Dervis, de Melo, and Robinson (1982). It incorporates additional features developed in recent years in research projects conducted at IFPRI. These features, which are of particular importance in developing countries, include household consumption of non-marketed (or “home”) commodities, explicit treatment of transaction costs for commodities that enter the market sphere, and a separation between producing activities and commodities that permits any activity to produce multiple commodities and any commodity to be produced by multiple activities. The paper also discusses the implementation of the model in GAMS (the General Algebraic Modeling System).¹

A self-extracting zip file is available which includes the GAMS files for the model, sample databases, simulations, solution reports, and a SAM aggregation program. In the GAMS code, the model is explicitly connected with a file for country data, including a “standard” SAM (*i.e.*, a SAM that follows the format that is required for the standard model), and a set of elasticities. Optionally, the user may provide quantity data for primary factors (for example labor types) that appear in the SAM. In the model code, this data set is used to define model parameter values in a manner that assures that the

¹ We assume that the reader has a basic familiarity with CGE modeling using GAMS. Brooke et al. (1998) is the basic reference on the GAMS software; it also includes a self-contained tutorial. The basics of GAMS-based CGE modeling are summarized in Robinson *et al.* (1999). Löfgren (2000a, 2000b) presents a set of hands-on exercises in CGE modeling with GAMS. Extensive treatments of CGE methods are found in Dervis, de Melo and Robinson (1982); Robinson (1989); Shoven and Whalley (1992); Dixon, Parmenter, Powell and Wilcoxon (1992); and Ginsburgh and Keyzer (1997). References to and examples of CGE-based analyses of food policy in developing countries are found in the section of the Trade and Macroeconomics Division on IFPRI's website (www.ifpri.org).

base solution to the model exactly reproduces the values in the SAM – the model is “calibrated” to the SAM. It is straightforward for users to develop new data sets for other applications.

The model and the accompanying GAMS code are written in a manner that gives the analyst considerable flexibility. He or she can choose between alternative treatments for macroeconomic balances and for factor markets. It is also possible to exclude various features that appear in the standard model, including home consumption and transactions costs. The country database to which the model should be applied can incorporate a wide range of policy tools as well as any desired degree of disaggregation of production activities, commodities, households, and enterprises. Flexibility in terms of model structure and the fact that model parameters are derived from an empirical database (which may be very detailed) permit the analyst to capture country-specific aspects of economic structure and functioning. Hence, although the paper provides a standardized framework for analysis, the analyst is not forced to make a “one-size-fits-all” assumption.

We consider this model and the accompanying computer code as work in progress and encourage readers and users to send us their comments.

The paper is organized as follows: Section 2 describes the standard SAM. Section 3 gives an overview of model features, followed by an equation-by-equation description in Section 4. Section 5 describes the structure of the GAMS files for the standard model and its database, and discusses how they may be used for policy analysis. The appendix includes the mathematical model statement in summary form, and core sections of the GAMS code for the model. The zip file includes the complete set of GAMS files.

2. THE SOCIAL ACCOUNTING MATRIX

A Social Accounting Matrix (SAM) is a comprehensive, economywide data framework, typically representing the economy of a nation.² More technically, a SAM is

² For general discussions of SAMs, see Pyatt and Round (1985), and Reinert and Roland-Holst (1997); for perspectives on SAM-based modeling, see Pyatt (1988), and Robinson and Roland-Holst (1988).

a square matrix in which each account is represented by a row and a column. Each cell shows the payment from the account of its column to the account of its row – the incomes of an account appear along its row, its expenditures along its column. The underlying principle of double-entry accounting requires that, for each account in the SAM, total revenue (row total) equals total expenditure (column total).³

Table 1 shows an aggregated SAM (with verbal explanations in place of numbers). With one exception, it has all the features required for implementation with the standard model. The exception is that, in the standard SAM, taxes have to be paid to tax accounts, disaggregated by tax type, each of which forwards its revenues to the core government account. The tax types are divided into direct taxes (on domestic non-government institutions and factors), commodity sales taxes, import taxes, export taxes, activity taxes, and value-added taxes. Also note that in the standard SAM payments are not permitted in the blank cells of Table 1. Any original SAM that includes such payments should be restructured before being implemented with the standard model.⁴

Table 2 shows a real-world standard SAM for Zimbabwe where the tax accounts are treated in the required manner.⁵ In addition, it has multiple accounts for activities, commodities, factors, and domestic non-government institutions – in each category, the GAMS code can handle any desired disaggregation, including having just a single account.

³ The GAMS program checks that the SAM that is entered is balanced, i.e., that for each account, the row and column totals are equal. If the absolute value of the sum of account imbalances exceeds a cutoff point, an optimization program is used to estimate a balanced SAM. The program, which minimizes the entropy distance of the cells of the estimated SAM from those of the initial SAM subject to the constraint that row and column totals are equal, is primarily intended to remove rounding errors. For SAM estimation in GAMS in a setting with substantial imbalances in raw data (not only rounding errors), see Robinson and El-Said (2000) and Robinson, Cattaneo, and El-Said (2001).

⁴ One common case would be payments from the government to factors (for the labor services provided by government employees). To restructure the SAM to work with the standard model, the preferred approach is to reallocate such payments to a commodity for government services that pays a government service activity which, in its turn, pays the labor account.

⁵ For other examples of SAMs that have the required structure, see the page for Data Sets on IFPRI's webpage (www.ifpri.org).

Table 1. Basic SAM structure used in CGE model

Receipts	Expenditures								
	Activities	Commodities	Factors	Households	Enterprises	Government	Savings-Investment	Rest of the World	TOTAL
Activities		marketed outputs		home-consumed outputs					Activity income (Gross Output)
Commodities	intermediate inputs	transactions costs		private consumption		government consumption	investment	exports	Demand
Factors	value-added							factor income from RoW	factor income
Households			factor income to households	inter-household transfers	surplus to households	transfers to households		transfers to households from RoW	household income
Enterprises			factor income to enterprises			transfers to enterprises		transfers to enterprises from RoW	enterprise income
Government	producer taxes, value-added tax	sales taxes, tariffs, export taxes	factor income to government, factor taxes	transfers to government, direct household taxes	surplus to government, direct enterprise taxes			transfers to government from RoW	government income
Savings-Investment				household savings	enterprise savings	government savings		foreign savings	savings
Rest of the World (RoW)		imports	factor income to RoW		surplus to RoW	government transfers to RoW			foreign exchange outflow
TOTAL	activity expenditures	supply	factor expenditures	household expenditures	enterprise expenditures	government expenditures	investment	foreign exchange inflow	

Table 2. Standard SAM for Zimbabwe, 1991 (Z\$ million)

	Activities					Commodities				Transactions costs			Factors			Households		Other institutions										TOTAL
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25			
Activities																												
1. Agric. large-scale						5,250																						5,250
2. Agric. Small-scale						670										685												1,355
3. Industry						17,859																						17,859
4. Transportation						8,263																						8,263
5. Other services						15,781																						15,781
Commodities																												
6. Agriculture	244	7	2,697	97												616	629					2,964			-30	7,223		
7. Industry	1,145	165	5,859	3,075	3,619											5,236	7,256	310				2,513	3,399	-494	32,083			
8. Transportation	38	29	176	183	282					3,444	1,689	986				605	662	169								8,263		
9. Other services	715	30	281	421	1,685											1,913	2,561	4,295						1,598	2,784	16,283		
Transactions costs																												
10. Domestic sales						657	2,788																					3,444
11. Imports						9	1,680																					1,689
12. Exports						580	406																					986
Factors																												
13. Labor	755	684	2,936	2,447	6,028																							12,851
14. Capital	1,719	260	5,386	1,950	3,524																							12,839
15. Land	458	137																										594
Households																												
16. Rural													1,605	1,979	594	259		5,526	1,113					102			11,179	
17. Urban													11,220	127					3,306	346							14,998	
Other institutions																												
18. Enterprise													10,733					1,209								11,942		
19. Government																		3,727				1,478	1,861	291			7,357	
20. Direct taxes																709		1,351	1,667								3,727	
21. Indirect taxes	43					524	188	546																				1,478
22. Import tariffs						10	1,800			51																	1,861	
23. Rest of world						48	7,550			450				26						535	418					9,027		
24. Sav-Inv																1,415	2,280	908	-504					1,559			5,658	
25. Stock change																										-525	-525	
TOTAL	5,250	1,355	17,859	8,263	15,781	7,223	32,083	8,263	16,283	3,444	1,689	986	12,851	12,839	594	11,179	14,998	11,942	7,357	3,727	1,478	1,861	9,027	5,658	-525			

Source: Thomas and Bautista (1999).

With regard to the structure of the standard SAM, it is important to note the following:

- The SAM distinguishes between accounts for “activities” (the entities that carry out production) and “commodities.” The receipts are valued at producers’ prices in the activity accounts and at market prices (including indirect commodity taxes and transactions costs) in the commodity accounts. The commodities are activity outputs, either exported or sold domestically, and imports. This activity-commodity separation is preferred since it permits activities to produce multiple commodities (for example, a dairy activity may produce the commodities cheese and milk) while any commodity may be produced by multiple activities (for example, activities for small-scale and large-scale maize production may both produce the same maize commodity). In the commodity columns, payments are made to domestic activities, the rest of the world, and various tax accounts (for domestic and import taxes). This treatment provides the data needed to model imports as perfect or imperfect substitutes vis-à-vis domestic production.⁶
- Trade flows are explicitly associated with transactions (trade and transportation) costs, also referred to as marketing margins. For each commodity, the SAM accounts for the costs associated with domestic, import, and export marketing. For domestic marketing of domestic output, the marketing margin represents the cost of moving the commodity from the producer to the domestic demander. For imports, it represents the cost of moving the commodity from the border to the domestic demander, while for exports, it shows the cost of moving the commodity from the producer to the border. The Zimbabwe SAM in Table 2 shows how these transactions costs appear in commodity and activity accounts in the standard SAM: A services activity, in Table 2 called transportation (account 4), produces a commodity (account 8) that, like other commodities, may be purchased for

⁶ In addition, our model code makes it possible to treat selected imports as separate, “non-comparable” commodities (not produced domestically). In the commodity rows, such import commodities receive payments from one or more domestic users. In the columns, these payments would be passed on to the accounts for the rest of the world, import marketing margins, and relevant taxes. The columns for this category of imports do not have any payments to domestic activities.

intermediate use by activities and final use by institutions. However, in addition, the transportation commodity receives payments from three special accounts, representing the transactions costs associated with domestic sales, imports, and exports (accounts 10-12). These special accounts are paid by the accounts for marketed agricultural and industrial commodities (accounts 6 and 7). This implies that the total value of each commodity includes these transactions costs. The standard model will also work with SAMs without this treatment of (and these accounts for) transactions costs.

- As noted, the government is disaggregated into a core government account and different tax accounts, one for each tax type. This disaggregation is often necessary since the economic interpretation of some payments otherwise often is ambiguous. In any given application, the SAM may exclude any (or all) of the individual tax accounts. In the SAM, payments between the government and other domestic institutions are reserved for transfers.
- The domestic non-government institutions consist of households and enterprises. The enterprises earn factor incomes (a reflection of ownership of capital and/or land). They may also receive transfers from other institutions. Their incomes are used for direct taxes, savings, and transfers to other institutions. As opposed to households, enterprises do not consume. Assuming that the relevant data are available, it is preferable to have one or more accounts for enterprises when these have tax obligations and a savings behavior that are independent of the household sector. It is important to disaggregate the enterprise sector in a manner that captures differences across enterprises in terms of tax rates, savings rates, and the shares of retained earnings that are received by different household types. For example, in some settings it may be appropriate to disaggregate enterprises into non-agricultural (with earnings from non-agricultural capital), small-scale agricultural enterprises (earnings from land and capital controlled by small farmers), and large-scale agricultural enterprises (earned from the land and capital of large farmers). Technically, the standard model requires that the SAM have at least one household account; enterprise accounts are not necessary.

- The SAM distinguishes between home consumption, which is activity-based, and marketed consumption by households, which is commodity-based. Home consumption, which in the SAM appears as household payments to activities, is valued at producer prices, *i.e.*, without marketing margins and the sales taxes that may be imposed on marketed commodities.⁷ Household consumption of marketed commodities appears as payments from household accounts to commodity accounts, the values of which include marketing margins and commodity taxes. The standard model also accepts a SAM without (explicit) home consumption.

3. OVERVIEW OF THE STANDARD CGE MODEL

The model explains all the payments that are recorded in the SAM. As a consequence, the model follows the SAM disaggregation of factors, activities, commodities, and institutions. It is written as a set of simultaneous equations, many of which are non-linear. There is no objective function. The equations define the behavior of the different actors. In part, this behavior follows simple rules captured by fixed coefficients (for example, *ad valorem* tax rates). For production and consumption decisions, behavior is captured by non-linear, first-order optimality conditions. The equations also include a set of constraints that have to be satisfied by the system as a whole but which are not necessarily considered by any individual actor. These constraints cover markets (for factors and commodities) and macroeconomic aggregates (balances for savings-investment, the government, and the current-account of the rest of the world).

This section gives an overview of the basic characteristics of the model. As opposed to the more detailed presentation of the model equations, it does not use any mathematical notation.

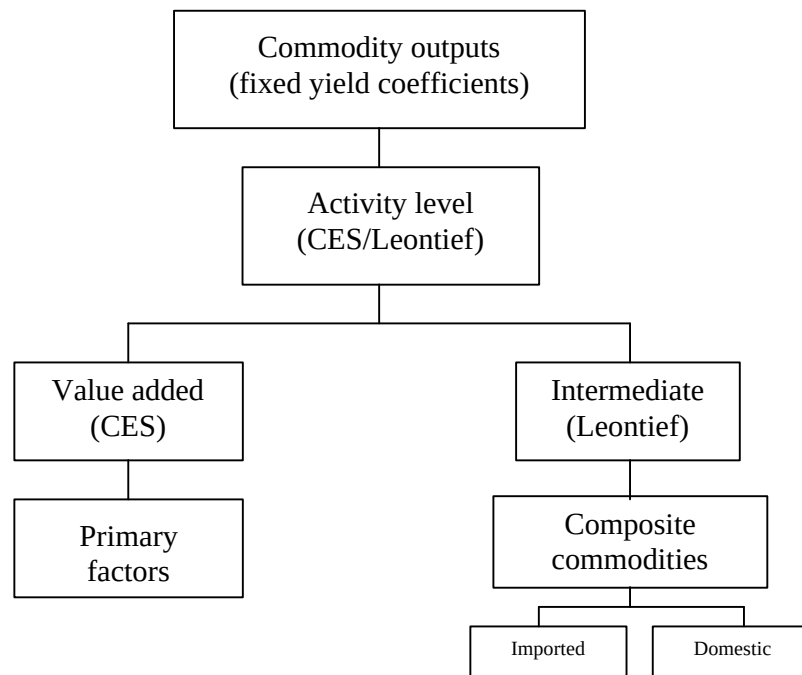
⁷ In the model, home consumption demand is for the commodity output(s) of the activities that, in the SAM, receive payments from households (*cf.* footnote 8 and equations 18 and 34 in section 4.3.)

3.1. ACTIVITIES, PRODUCTION, AND FACTOR MARKETS

Each producer (represented by an activity) is assumed to maximize profits, defined as the difference between revenue earned and the cost of factors and intermediate inputs. Profits are maximized subject to a production technology, the structure of which is shown in Figure 1. At the top level, the technology is specified by a CES or, alternatively, a Leontief function of the quantities of value-added and aggregate intermediate input. The Leontief alternative is the default. The CES alternative may be preferable in particular sectors if empirical evidence suggests that available techniques permit the aggregate mix between value-added and intermediate inputs to vary. Value-added is itself a CES function of primary factors whereas the aggregate intermediate input is a Leontief function of disaggregated intermediate inputs.

Each activity produces one or more commodities according to fixed yield coefficients. (As noted, any commodity may be produced by more than one activity.) The

Figure 1—Production technology



revenue of the activity is defined by the level of the activity, yields, and commodity prices at the producer level.

As part of its profit-maximizing decision, each activity uses a set of factors up to the point where the marginal revenue product of each factor is equal to its wage (also called factor price or rent). Factor wages may differ across activities, not only when the market is segmented but also for mobile factors. In the latter case, the model incorporates discrepancies that stem from exogenous causes (for example wage differences across activities due to considerations such as status, comfort, or health risks).

The user can choose between alternative factor market closures (mechanisms for equilibrating supplies and demands in factor markets). According to the default closure, the quantity supplied of each factor is fixed at the observed level. An economy-wide wage variable is free to vary to assure that the sum of demands from all activities equal the quantity supplied. Each activity pays an activity-specific wage that is the product of the economy-wide wage and an activity-specific wage (distortion) term. For the default closure, the latter terms are fixed.

Alternatively, it is possible to assume that a factor is unemployed and the real wage is fixed. This assumption may, for example, be appropriate in settings where there is considerable unemployment for a given labor category. Compared to the default closure, the only change is that the economy-wide wage variable is fixed (or exogenized) while the supply variable is “flexed” (or endogenized). Each activity is free to hire any desired quantity at its fixed, activity-specific wage (which, implicitly, is indexed to the model numéraire). In this setting, the supply variable is superfluous; it merely records the total quantity demanded.

Under a third closure, the factor market is segmented and each activity is forced to hire the observed, base-year quantity – the factor is activity-specific. This closure may be preferred in short-run analysis and/or when there are significant quality differences between the units of a factor that are used in different activities, for example units of non-agricultural capital used in different industrial and service activities. For this case, the quantities of activity-specific factor demands and the economy-wide wage are fixed while the activity-specific wage terms and the supply variables are flexible.

3.2. INSTITUTIONS

In the model, institutions are represented by households, enterprises, the government, and the rest of the world.

The households (disaggregated as in the SAM) receive income from the factors of production (directly or indirectly, via the enterprises), and transfers from other institutions. Transfers from the rest of the world to households are fixed in foreign currency. (All transfers between the rest of the world and domestic institutions and factors are fixed in foreign currency.) The households use their income to pay direct taxes, save, consume, and make transfers to other institutions. In the basic model version, direct taxes and transfers to other domestic institutions are defined as fixed shares of household income whereas the savings share is flexible for selected households. The treatment of direct tax and savings shares is related to the choice of closure rule for the government and savings-investment balances. (This topic is discussed below in Section 3.4). The income that remains (after taxes, savings, and transfers to other institutions) is spent on consumption.

Household consumption covers marketed commodities, purchased at market prices that include commodity taxes and transactions costs, and home commodities, which are valued at activity-specific producer prices.⁸ Household consumption is allocated across different commodities (both market and home commodities) according to Linear Expenditure System (LES) demand functions.

Instead of being paid directly to the households, factor incomes may be paid to one or more enterprises. Enterprises may also receive transfers from other institutions. Enterprise incomes are allocated to direct taxes, savings, and transfers to other institutions. Enterprises do not consume. Apart from this, the payments to and from enterprises are modeled in the same way as the same payments to and from households.

⁸ In the standard SAM, home consumption is only disaggregated by activity and household, not by commodity, activity, and household. When households consume out of activities that produce multiple outputs, extraneous, non-SAM data is needed to allocate home consumption across the commodities produced by each relevant multiple-output activity.

The government collects taxes and receives transfers from other institutions. In the basic model version, all taxes are at fixed *ad valorem* rates. The government uses this income to purchase commodities for its consumption and for CPI-indexed transfers to other institutions. In the basic model version, government consumption is fixed in real (quantity) terms whereas government transfers to domestic institutions (households and enterprises) are CPI-indexed. Government savings (the difference between government income and spending) is a flexible residual.

The rest of the world is the only remaining institution. As noted, transfer payments from the rest of the world and domestic institutions and factors are all fixed in foreign currency. Commodity trade with the rest of the world is discussed in the following section. Foreign savings (or the current account deficit) is the difference between foreign currency spending and receipts.

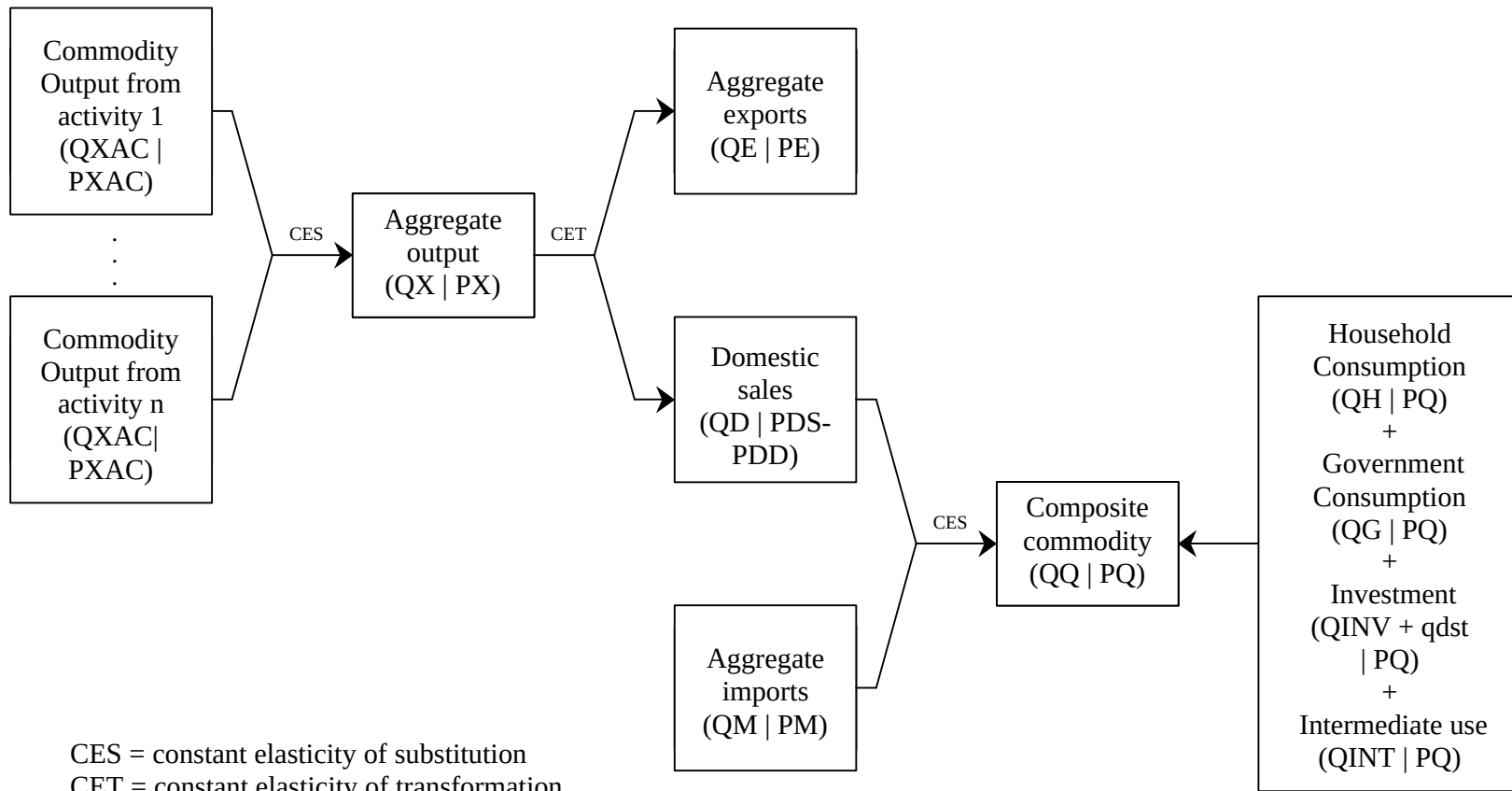
Section 3.4 discusses the rules for clearing the macroeconomic balances (the macro closures), *i.e.*, how equilibrium is achieved in the balances for the government, the rest of the world, and the savings-investment account (where institutional savings are aggregated and allocated to domestic investment).

3.3. COMMODITY MARKETS

With the exception of home-consumed output, all commodities (domestic output and imports) enter markets. Figure 2 shows the physical flows for marketed commodities and associated quantity and price variables as defined in the model equations discussed in section 4 below.

Domestic output may be sold in the market or consumed at home. For marketed output, the first stage in the chain consists of generating aggregated domestic output from the output of different activities of a given commodity. These outputs are imperfectly substitutable, for example as a result of differences in timing, quality, and location between different activities. A Constant-Elasticity-of-Substitution (CES) function is used

Figure 2. Flows of marketed commodities



as aggregation function. The demand for the output of each activity is derived from the problem of minimizing the cost of supplying a given quantity of aggregated output subject to this CES function. Activity-specific commodity prices serve the role of clearing the implicit market for each disaggregated commodity.

At the next stage, aggregated domestic output is allocated between exports and domestic sales on the assumption that suppliers maximize sales revenue for any given aggregate output level, subject to imperfect transformability between exports and domestic sales, expressed by a Constant-Elasticity-of-Transformation (CET) function. In the international markets, export demands are infinitely elastic at given world prices. The price received by domestic suppliers for exports is expressed in domestic currency and adjusted for the transactions cost (to the border) and export taxes (if any). The supply price for domestic sales is equal to the price paid by domestic demanders minus the transactions cost of domestic marketing (from the supplier to the demander) per unit of domestic sales. If the commodity is not exported, total output is passed to the domestic market.

Domestic demand is made up of the sum of demands for household consumption, government consumption, investment (the determination of which is discussed below), intermediate inputs, and transactions (trade and transportation) inputs.

To the extent that a commodity is imported, all domestic market demands are for a composite commodity made up of imports and domestic output, the demands for which are derived on the assumption that domestic demanders minimize cost subject to imperfect substitutability. This is also captured by a CES aggregation function.⁹ Total market demand is directed to imports for commodities that lack domestic production and to domestic output for non-imported commodities.

The derived demands for imported commodities are met by international supplies that are infinitely elastic at given world prices. The import prices paid by domestic demanders also include import tariffs (at fixed *ad valorem* rates) and the cost of a fixed

⁹ This function is also referred to as an Armington function, named after Paul Armington who introduced imperfect substitutability between imports and domestic commodities in economic models (Armington 1969).

quantity of transaction services per import unit (which cover the cost of moving the commodity from the border to the demander).¹⁰ Similarly, the derived demand for domestic output is met by domestic suppliers. The prices paid by the demanders include the cost of transaction services (in this case reflecting that the commodity was moved from the domestic supplier to the domestic demander). The prices received by domestic suppliers are net of this transactions cost. Flexible prices equilibrate demands and supplies of domestically marketed domestic output.

The assumptions of imperfect transformability (between exports and domestic sales of domestic output) and imperfect substitutability (between imports and domestically sold domestic output) permit the model to better reflect the empirical realities of most countries (in comparison with the alternative assumptions of perfect substitutability and transformability). The assumptions that are used give the domestic price system a degree of independence from international prices and avoid unrealistic export and import responses to economic shocks. At the disaggregated commodity level, these assumptions allow for a continuum of tradability and two-way trade, which commonly is observed even at very fine levels of disaggregation.

3.4. MACROECONOMIC BALANCES

The model includes three macroeconomic balances: the (current) government balance, the external balance (the current account of the balance of payments, which includes the trade balance), and the savings-investment balance. In the GAMS code, the user has the option to choose among a relatively large number of pre-programmed alternative closure rules for these balances. The closures are summarized in Table 3.¹¹

¹⁰ Note that these transactions costs are not *ad valorem* – the rates (the ratio between the margin and the price without the margin) change when there are changes in the prices of transactions services and/or the commodities that are marketed.

¹¹ Macro closures of CGE models is a contentious topic with a large literature. For summaries, see Robinson (1989), Rattsø(1982), and Taylor (1990).

Table 3. Alternative closure rules for macro system constraints*

Constraint		
Government	Rest of World	Savings-Investment
GOV-1: Flexible government savings; fixed direct tax rates	ROW-1: Fixed foreign savings; flexible real exchange rate	SI-1: Fixed capital formation; uniform MPS** point change for selected institutions
GOV-2: Fixed government savings; uniform direct tax rate point change for selected institutions	ROW-2: Flexible foreign savings; fixed real exchange rate	SI-2: Fixed capital formation; scaled MPS for selected institutions
GOV-3: Fixed government savings; scaled direct tax rates for selected institutions		SI-3: Flexible capital formation; fixed MPS for all non-government institutions
		SI-4: Fixed investment and government consumption absorption shares (flexible quantities); uniform MPS point change for selected institutions
		SI-5: Fixed investment and government consumption absorption shares (flexible quantities); scaled MPS for selected institutions

* For the specified closure rules, the choice for one of the three constraints does not constrain the choice for the other two constraints. ** MPS = marginal propensity to save.

For the government balance, the default closure (GOV-1) is that government savings (the difference between current government revenues and current government expenditures) is a flexible residual while all tax rates are fixed. Under two alternative government closures, the direct tax rates of domestic institutions (households and enterprises) are adjusted endogenously to generate a fixed level of government savings. For the first of these alternative closures (GOV-2), the base-year direct tax rates of selected domestic non-government institutions (households and enterprises) are adjusted endogenously by the same number of percentage points. For the second (GOV-3), the

rates of selected institutions are multiplied by a flexible scalar.¹² For these three government closures, government consumption is fixed, either in real terms or as a share of nominal absorption (depending on the treatment of the savings-investment balance, discussed below). (I.e., we do not specify a closure where government savings and direct tax rates are both fixed and government consumption is the adjusting variable.)

For the external balance (which is expressed in foreign currency), the default closure (ROW-1) is that the real exchange rate is flexible while foreign savings (the current account deficit) is fixed. Given that all other items in the external balance (transfers between the rest of the world and domestic institutions) are fixed, the trade balance is also fixed. If, *ceteris paribus*, foreign savings are below the exogenous level, a depreciation of the real exchange rate would correct this situation by simultaneously (i) reducing spending on imports (a fall in import quantities at fixed world prices); and (ii) increasing earnings from exports (an increase in export quantities at fixed world prices). Under an alternative closure (ROW-2), the real exchange rate (indexed to the model numéraire) is fixed while foreign savings (and the trade balance) is flexible.¹³

For the savings-investment balance, closures are either investment-driven (the value of savings adjusts) or savings-driven (the value of investment adjusts). The default closure (SI-1) is investment-driven. Real investment quantities are fixed. In order to generate savings that equal the cost of the investment bundle, the base-year savings rates of selected non-government institutions are adjusted by the same number of percentage points.

Four additional closures are specified. The first alternative (SI-2) is also investment-driven. It differs from the default in that, instead of adjusting base-year

¹² The difference between these two closures in terms of simulated changes in post-tax incomes may be substantial, as illustrated by an example with two institutions, an enterprise and a household that, under base conditions, both have incomes of 200 and face direct tax rates of 20% and 10%, respectively. Assume that total direct tax collection has to increase from 60 to 90 in order to reach a fixed level of government savings (assuming, for simplicity, no income changes). Under the first closure, the rates would increase by 7.5 percentage points for both institutions, to 27.5% for the enterprise and 17.5% for the household. The payments would increase by 15 for both. Under the second closure, the new tax rates would be 30% and 15% (multiplying both base rates by 1.5), respectively. The tax payments increase by 20 for the enterprise and by 10 for the household.

¹³ For a discussion of the real exchange rate in neoclassical, trade-focused CGE models, see Devarajan, Lewis, and Robinson (1993).

savings rates by a fixed number of percentage points, the rates of selected institutions are multiplied by a scalar (*cf.* the above discussion of the treatment of direct tax rates under alternative government closures). The second (SI-3) is savings-driven. All non-government savings rates are fixed. The quantity of each commodity in the investment bundle is multiplied by a flexible scalar so as to assure that the investment cost will be equal to the savings value.

The last two alternatives (SI-4 and SI-5) are types of “balanced” closures, which may be viewed as variants of investment-driven closures (although they also impose an adjustment rule for government consumption). Under these, adjustments in absorption are spread across all of its components (household consumption, investment, and government consumption).¹⁴ The nominal absorption shares of investment and government consumption are fixed at base levels (although this could be generalized). (Except for SI-4 and SI-5, government consumption is fixed in real terms.) Given this specification, the residual share for household consumption is also fixed. For the first balanced closure (SI-4), the savings rates of selected institutions are adjusted by an equal number of percentage points (*cf.* SI-1). For the second balanced closure (SI-5), the savings rates of selected institutions are scaled so as to generate enough savings to finance investment (*cf.* SI-2). The balanced closures are compatible with any combination of the pre-programmed closures for the government and the rest of the world.

The appropriate choice between the different macro closures depends on the context of the analysis. Given that this is a single-period model, a closure combining fixed foreign savings, fixed real investment, and fixed real government consumption may be preferable for simulations that explore the equilibrium welfare changes of alternative policies. Such a closure avoids the misleading welfare effects that appear when foreign savings and real investment change in simulations with a single-period model – *ceteris paribus*, for the simulated period, increases in foreign savings and decreases in investment raise household welfare (and vice versa for decreases in foreign savings and

¹⁴ Under the other investment-driven closures, the quantities of investment and government consumption are both fixed. Hence, household consumption is the only part of absorption that adjusts (in response to changes in savings rates). Under the savings-driven closure, the bulk of the adjustment is carried by investment.

increases in investment). This result is misleading since the analysis does not capture welfare losses in later periods that arise from a larger foreign debt and a smaller capital stock. With regard to government consumption, the model does not capture its direct and indirect welfare contributions; to avoid misleading results, it is also preferable to keep this variable fixed.

On the other hand, if the analysis aims at capturing the likely welfare effects of an exogenous shock or policy change in a given (historical, current, or future) setting, perhaps in order to explore the role for complementary policies, it may be preferable to impose a closure that more closely mimics the real world, often involving simultaneous adjustments in the three components of absorption. Under these circumstances, a macro scenario that incorporates a balanced closure is a useful option.

In addition, it is often informative to explore the impact of any experiment under a set of alternative macro closures. The results often provide important insights into the real-world trade-offs that are associated with alternative macroeconomic adjustment patterns.

4. MATHEMATICAL MODEL STATEMENT

In this section, the mathematical model statement is presented, equation by equation. In its mathematical form, the model is a system of simultaneous, non-linear equations. The model is square – the number of equations is equal to the number of variables. In this class of models, this is a necessary (but not a sufficient) condition for the existence of a unique solution. In the statement, the equations are divided into four blocks: prices; production and trade; institutions; and system constraints. New items (sets, parameters, and variables) are defined the first time that they appear in the equations. The notational principles are summarized in Table 4. In addition, parameter and variable names have been chosen to facilitate interpretation; most importantly, commodity and factor quantities start with q , commodity prices with p , and factor prices with w .

Table 4. Notational principles

<u>Item</u>	<u>Notation</u>
Endogenous variables	upper-case Latin letters without a bar
Exogenous variables	upper-case Latin letters with a bar
Parameters	lower-case Latin letters (with or without a bar) or lower-case Greek letters (with or without superscripts)
Set indices	Lower-case Latin letters as subscripts to variables and parameters

Note: Exogenous variables are fixed in the basic model version but may be endogenous in versions with different treatments of macro or factor market closures.

4.1. PRICE BLOCK

The price system of the model is rich, primarily as a result of the assumed quality differences between commodities of different origins and destinations (exports, imports, and domestic outputs used domestically). The price block consists of equations in which endogenous model prices are linked to other prices (endogenous or exogenous) and non-price model variables.

Import price

$$PM_c = pwm_c \cdot (1 + tm_c) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c', c} \quad c \in CM \quad (1)$$

$$\begin{bmatrix} \text{import} \\ \text{price} \\ (LCU) \end{bmatrix} = \begin{bmatrix} \text{import} \\ \text{price} \\ (FCU) \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust -} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ (LCU per \\ FCU) \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{import unit} \end{bmatrix}$$

where

$c \in C$ set of commodities (also referred to as c' and C')
 $c \in CM (\subset C)$ set of imported commodities

$c \in CT (\subset C)$	set of domestic trade inputs (distribution commodities)
PM_c	import price in LCU (local-currency units; including transactions costs)
pwm_c	c.i.f. import price in FCU (foreign-currency units)
tm_c	import tariff rate
EXR	exchange rate (LCU per FCU)
PQ_c	composite commodity price (including sales tax and transactions costs)
$icm_{c'c}$	quantity of commodity c' as trade input per imported unit of c

The import price in LCU (local currency units) is the price paid by domestic users for imported commodities (exclusive of the sales tax). Equation (1) states that it is a transformation of the world price of these imports, considering the exchange rate and import tariffs plus transactions cost (the cost of trade inputs needed to move the commodity from the border to the demander) per unit of the import. For all commodities, the market price paid by domestic commodity demanders is the composite price, PQ ; in this equation, PQ only applies to payments for trade inputs. The domain of the equation is the set of imported commodities (a subset of the commodity set) – the model includes one equation like (1) for every imported commodity.

Note that the notational principles make it possible to distinguish between variables (upper-case Latin letters) and parameters (lower-case Latin letters). This means that the exchange rate and the domestic import price are flexible, while the tariff rate and the world import price are fixed. The fixedness of the latter price stems from the “small-country” assumption – for all its imports, the assumed share of world trade for the modeled country is so small that it faces an infinitely elastic supply curve at the prevailing world price.

Export price

$$PE_c = pwe_c \cdot (1 - te_c) \cdot EXR - \sum_{c' \in CT} PQ_c \cdot icm_{c'c} \quad c \in CE \quad (2)$$

$$\begin{bmatrix} \text{export} \\ \text{price} \\ \text{(LCU)} \end{bmatrix} = \begin{bmatrix} \text{export} \\ \text{price} \\ \text{(FCU)} \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust -} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ \text{(LCU per} \\ \text{FCU)} \end{bmatrix} - \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{export unit} \end{bmatrix}$$

where

$c \in CE(\subset C)$	set of exported commodities (with domestic production)
PE_c	export price (LCU)
pwe_c	f.o.b. export price (FCU)
te_c	export tax rate
$ice_{c'c}$	quantity of commodity c' as trade input per exported unit of c

The export price in LCU is the price received by domestic producers when they sell their output in export markets. This equation is similar in structure to the import price definition. The main difference is that the tax and the cost of trade inputs reduce the price received by the domestic producers of exports (instead of adding to the price paid by domestic demanders of imports). The domain of the equation is the set of exported commodities, all of which are produced domestically.¹⁵

Demand price of domestic non-traded goods

$$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c'c}$$

$$\begin{bmatrix} \text{domestic} \\ \text{demand} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{domestic} \\ \text{supply} \\ \text{price} \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{unit of} \\ \text{domestic sales} \end{bmatrix} \quad c \in CD \quad (3)$$

where

$c \in CD(\subset C)$	set of commodities with domestic sales of domestic output
PDD_c	demand price for commodity produced and sold domestically
PDS_c	supply price for commodity produced and sold domestically
$icd_{c'c}$	quantity of commodity c' as trade input per unit of c produced and sold domestically

The model includes distinct prices for domestic output that is used domestically. In the presence of transaction costs, it is necessary to distinguish between prices paid by

¹⁵ The model does not include any commodities that are imported for immediate re-export. As long as such trade uses domestic factors (and, possibly, intermediate inputs), it can be handled without any changes in model structure by including in the SAM an activity that imports a non-produced commodity, and exports all of its output.

demanders and received by suppliers. Equation (3) defines the demand prices as the supply price plus the cost of trade inputs per unit of domestic sales of the commodity in question.

Absorption

$$PQ_c \cdot (1 - tq_c) \cdot QQ_c = PDD_c \cdot QD_c + PM_c \cdot QM_c$$

$$\left[\begin{array}{c} \text{absorption} \\ \text{(at demand} \\ \text{prices net of} \\ \text{sales tax)} \end{array} \right] = \left[\begin{array}{c} \text{domestic demand price} \\ \text{times} \\ \text{domestic sales quantity} \end{array} \right] + \left[\begin{array}{c} \text{import price} \\ \text{times} \\ \text{import quantity} \end{array} \right] \quad c \in (CD \cup CM) \quad (4)$$

where

QQ_c	quantity of goods supplied to domestic market (composite supply)
QD_c	quantity sold domestically of domestic output
QM_c	quantity of imports of commodity
tq_c	rate of sales tax (as share of composite price inclusive of sales tax)

Absorption is total domestic spending on a commodity at domestic demander prices. In Equation (4) it is defined exclusive of the sales tax. Absorption is expressed as the sum of spending on domestic output and imports at the demand prices, PDD and PM . (The prices PDD and PM include the cost of trade inputs but exclude the commodity sales tax; cf. Equations 1 and 3.)

The equation as a whole applies to all commodities that are imported and/or have domestic sales of domestic output (the union of the sets CD and CM). It does not apply to commodities for which the entire output volume is exported. Each of the two terms on the right-hand side only applies to its relevant set (CD and CM , respectively). In the GAMS code, PM and QM are fixed at zero for commodities that are not elements in the set CM ; similarly PDD and QD are fixed at zero for commodities that are not elements in the set CD . This approach is followed throughout: all variables that should be excluded from the model are fixed at zero. Note that division by $(1 - tq)$ or $(1 - tq) \cdot QQ$, would transform

this equation into explicit definitions of absorption at market prices or definitions of the composite price (the price paid by domestic demanders, inclusive of the sales tax).

Marketed Output Value

$$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c$$

$$\begin{bmatrix} \text{producer price} \\ \text{times marketed} \\ \text{output quantity} \end{bmatrix} = \begin{bmatrix} \text{domestic supply price} \\ \text{times} \\ \text{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \text{export price} \\ \text{times} \\ \text{export quantity} \end{bmatrix} \quad c \in CX \quad (5)$$

where

PX_c aggregate producer price for commodity
 QX_c aggregate quantity of domestic output of commodity
 QE_c quantity of exports
 $c \in CX (\subset C)$ set of commodities with domestic output

For each domestically produced commodity, the marketed output value at producer prices is stated as the sum of the values of domestic sales and exports.¹⁶ Domestic sales and exports are valued at the prices received by the suppliers, PDS and PE , both of which have been adjusted downwards to account for the cost of trade inputs (cf. Equations 2 and 3).

The domain limitation to domestically produced commodities (the elements in the set CX) has to be stated explicitly given that the model includes a category of imported commodities without domestic production. The domestic part only applies to elements in CD whereas the export part only applies to elements in CE . (In the GAMS code, the variables PE and QE are fixed at zero for commodities that are not elements in the set CE .) PX and QX are referred to as “aggregate” values since they may apply to an aggregation of output from different domestic producers of the same commodity. By dividing through by QX , this equation could be rewritten as an explicit definition of PX .

¹⁶ This value excludes the value of home-consumed output.

Activity Price

$$PA_a = \sum_{c \in C} PXAC_{ac} \cdot q_{ac}$$

$$\begin{bmatrix} \text{activity} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{producer prices} \\ \text{times yields} \end{bmatrix} \quad a \in A \quad (6)$$

where

$a \in A$	set of activities
PA_a	activity price (gross revenue per activity unit)
$PXAC_{ac}$	producer price of commodity c for activity a
q_{ac}	yield of output c per unit of activity a

The gross revenue per activity unit (the activity price) is the return from selling the output (or outputs) of the activity, defined as yields per activity unit multiplied by activity-specific commodity prices, summed over all commodities (to allow for the fact that activities may produce multiple commodities).

Aggregate intermediate input price

$$PINTA_a = \sum_{c \in C} PQ_c \cdot ica_{ca}$$

$$\begin{bmatrix} \text{aggregate} \\ \text{intermediate} \\ \text{input price} \end{bmatrix} = \begin{bmatrix} \text{intermediate input cost} \\ \text{per unit of aggregate} \\ \text{intermediate input} \end{bmatrix} \quad a \in A \quad (7)$$

where

$PINTA_a$	aggregate intermediate input price for activity a
ica_{ca}	quantity of c per unit of aggregate intermediate input a

The activity-specific aggregate intermediate input price shows the cost of disaggregated intermediate inputs per unit of aggregate intermediate input. It depends on composite commodity prices and intermediate input coefficients (which show the quantity of input commodity c per unit of aggregate intermediate input, not per unit of output).

Activity revenue and costs

$$PA_a \cdot (1 - ta_a) \cdot QA_a = PVA_a \cdot QVA_a + PINTA_a \cdot QINTA_a$$

$$\left[\begin{array}{c} \text{activity price} \\ \text{(net of taxes)} \\ \text{times activity level} \end{array} \right] = \left[\begin{array}{c} \text{value-added} \\ \text{price times} \\ \text{quantity} \end{array} \right] + \left[\begin{array}{c} \text{aggregate} \\ \text{intermediate} \\ \text{input price times} \\ \text{quantity} \end{array} \right] \quad a \in A \quad (8)$$

where

ta_a	tax rate for activity
QA_a	quantity (level) of activity
QVA_a	quantity of (aggregate) value-added
$QINTA_a$	quantity of aggregate intermediate input
PVA_a	price of (aggregate) value-added

For each activity, total revenue (net of taxes) is fully exhausted by payments for value-added and intermediate inputs. Given the above definitions of PA and PINTA, this equation implicitly defines the value-added price, PVA.

Consumer price index

$$\overline{CPI} = \sum_{c \in C} PQ_c \cdot cwtsc$$

$$\left[\overline{CPI} \right] = \left[\begin{array}{c} \text{prices times} \\ \text{weights} \end{array} \right] \quad (9)$$

where

$cwtsc$	weight of commodity c in the consumer price index
$\overline{CPI}$	consumer price index (exogenous variable)

Producer price index for non-traded market output

$$DPI = \sum_{c \in C} PDS_c \cdot dwts_c$$

$$\left[\begin{array}{c} \text{Producer price index} \\ \text{for non-traded outputs} \end{array} \right] = \left[\begin{array}{c} \text{prices times} \\ \text{weights} \end{array} \right] \quad (10)$$

where

$dwts_c$	weight of commodity c in the producer price index
DPI	producer price index for domestically marketed output

Equations 9 and 10 define the consumer price index and the producer price index for domestically marketed output. The CPI is fixed and functions as the numéraire in the basic model version; alternatively, the DPI may be fixed. A numéraire is required since the model is homogeneous of degree zero in prices – a doubling of the value of the numéraire would double all prices but leave all real quantities unchanged. All simulated price and income changes should be interpreted as changes vis-à-vis the numéraire price index.

4.2. PRODUCTION AND TRADE BLOCK

This equation block covers domestic production and input use; the allocation of domestic output to home consumption, the domestic market, and exports; the aggregation of supply to the domestic market (from imports and domestic output sold domestically); and the definition of the demand for trade inputs that is generated by the distribution process.

Production is carried out by activities that are assumed to maximize profits subject to their technology, taking prices (for their outputs, intermediate inputs, and factors) as given, *i.e.*, acting in a perfectly competitive setting. The model includes the first-order conditions for profit-maximization by producers. As noted in the preceding section (see Figure 1), two alternative specifications are permitted at the top level of the nested technology: the activity level is either a CES or a Leontief function of the quantities of value-added and aggregate intermediate input use.¹⁷

¹⁷ For the alternative with CES technology at the top of the technology nest, the profit-maximization problem (which applies to each relevant activity, a) is:

CES technology: Activity Production Function

$$QA_a = a_a^a \cdot \left(d_a^a \cdot QVA_a^{-r_a^a} + (1-d_a^a) \cdot QINTA_a^{-r_a^a} \right)^{\frac{1}{r_a^a}} \quad a \in ACES \quad (11)$$

$$\begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix} = CES \begin{bmatrix} \text{quantity of aggregate value-added,} \\ \text{quantity aggregate intermediate input} \end{bmatrix}$$

CES technology: Value-Added – Intermediate-Input Ratio

$$\frac{QVA_a}{QINTA_a} = \left(\frac{PINTA_a \cdot d_a^a}{PVA_a \cdot (1-d_a^a)} \right)^{\frac{1}{1+r_a^a}} \quad a \in ACES \quad (12)$$

$$\begin{bmatrix} \text{value-added} - \\ \text{intermediate-} \\ \text{input quantity} \\ \text{ratio} \end{bmatrix} = f \begin{bmatrix} \text{intermediate-input} \\ - \text{value-added} \\ \text{price ratio} \end{bmatrix}$$

where

$a \in ACES (\subset A)$	set of activities with a CES function at the top of the technology nest
a_a^a	efficiency parameter in the CES activity function
d_a^a	CES activity function share parameter
r_a^a	CES activity function exponent

The user specifies the activities (if any) that belong to the set ACES. (If not in ACES, an activity belongs to the set ALEO, which is introduced below). Activities in ACES, have CES technology at the top level of the technology nest, *i.e.*, the activity level is a CES function of value-added and aggregate intermediate input use (Equation 11). The optimal mix of intermediate inputs and value-added is a function of the relative

$$\text{maximize } PA_a \cdot (1-ta_a) \cdot QA_a - \sum_{c \in C} PQ_c \cdot QINT_c - \sum_{f \in F} WF_f \cdot WFDIST_{fa} \cdot QF_{fa} \quad \text{subject to equations 11, 15, and 17;}$$

equations 7-8, 11-12, and 15-17 are the first-order conditions. For the alternative with Leontief technology at the top, the profit-maximization problem includes equations 13-14 among its constraints, but excludes equation 11; the Leontief first-order conditions include equations 13-14 instead of equations 11-12. In both *optimization* problems, all quantities are variables whereas the decision makers view all other items as parameters or exogenous variables (including all prices and wages).

prices of value-added and the aggregate intermediate input (Equation 12).¹⁸ (Below, in Equation 18, the activity level determines the quantity of commodity outputs produced by each activity.) The exponent, r_a^a , is a transformation of the elasticity of substitution between value-added and the aggregate intermediate input: the higher this elasticity, the smaller the value of r_a^a and the larger the optimal change in the ratios between the quantities of value-added and the intermediate input aggregate in response to changes in their relative prices.¹⁹

Leontief technology: Demand for aggregate value-added

$$QVA_a = iva_a \cdot QA_a$$

$$\left[\begin{array}{c} \text{demand for} \\ \text{value-added} \end{array} \right] = f \left[\begin{array}{c} \text{activity} \\ \text{level} \end{array} \right] \quad a \in ALEO \quad (13)$$

Leontief technology: Demand for aggregate intermediate input

$$QINTA_a = inta_a \cdot QA_a$$

$$\left[\begin{array}{c} \text{demand for aggregate} \\ \text{intermediate input} \end{array} \right] = f \left[\begin{array}{c} \text{activity} \\ \text{level} \end{array} \right] \quad a \in ALEO \quad (14)$$

where

$a \in ALEO(\subset A)$	set of activities with a Leontief function at the top of the technology nest
iva_a	quantity of value-added per activity unit
$inta_a$	quantity of aggregate intermediate input per activity unit

¹⁸ In general, when writing non-linear equations to be solved numerically, it is good practice to avoid division by a variable that the solver treats as possibly going to zero. Accordingly, in the GAMS version of Equation 12, $QINTA$ has been moved to the right-hand side. Parallel adjustments have been made in Equations 22 and 25.

¹⁹ For CES functions, $s = \frac{1}{1+r}$, where s is the elasticity of substitution and r the exponent.

For the alternative model version with a Leontief function at the top level of the technology nest, Equations 11-12 are replaced by Equations 13-14 where the demands for value-added and the aggregate intermediate inputs are defined as Leontief functions of the activity level. Each activity is an element in *ACES* or *ALEO*, not both.

Value-added and factor demands

$$QVA_a = a_a^{va} \cdot \left(\sum_{f \in F} d_{fa}^{va} \cdot QF_{fa}^{-r_a^{va}} \right)^{-\frac{1}{r_a^{va}}} \quad a \in A \quad (15)$$

$$\left[\begin{array}{c} \text{quantity of aggregate} \\ \text{value-added} \end{array} \right] = CES \left[\begin{array}{c} \text{factor} \\ \text{inputs} \end{array} \right]$$

Factor demand

$$W_f \cdot \overline{WFDIST}_{fa} = PVA_a \cdot (1 - tva_a) \cdot QVA_a \cdot \left(\sum_{f \in F'} d_{fa}^{va} \cdot QF_{fa}^{-r_a^{va}} \right)^{-1} \cdot d_{fa}^{va} \cdot QF_{fa}^{-r_a^{va}-1} \quad \begin{array}{l} a \in A \\ f \in F \end{array} \quad (16)$$

$$\left[\begin{array}{c} \text{marginal cost of} \\ \text{factor } f \text{ in activity } a \end{array} \right] = \left[\begin{array}{c} \text{marginal revenue product} \\ \text{of factor } f \text{ in activity } a \end{array} \right]$$

where

$f \in F (= F')$	set of factors
tva_a	rate of value-added tax for activity a
a_a^{va}	efficiency parameter in the CES value-added function
d_{fa}^{va}	CES value-added function share parameter for factor f in activity a
QF_{fa}	quantity demanded of factor f from activity a
r_a^{va}	CES value-added function exponent
WF_f	average price of factor
$\overline{WFDIST}_{fa}$	wage distortion factor for factor f in activity a (exogenous variable)

Equation 15 states that, for each activity, the quantity of value-added is a CES function of disaggregated factor quantities. According to Equation 16, activities demand factors at the point where the marginal cost of each factor (defined on the left-hand side

as the activity-specific factor price) is equal to the marginal revenue product (net of intermediate input costs) of the factor. In the GAMS code, the domain of Equation 16 is limited to the factor-activity combinations that appear in the base-year SAM; similar domain restrictions apply to other equations that are defined over mappings between multiple indices (for example Equation 17). The exponent, r_a^{va} , is a transformation of the elasticity of factor substitution: the higher this elasticity, the smaller the value of r_a^{va} and the larger the optimal change in the ratios between different factor quantities in response to changes in relative factor prices. (Cf. footnote 17 above.)

The fact that the average factor price is an endogenous variable while the activity-specific “wage-distortion” factor is exogenous reflects the treatment of factor markets in the basic model version (see Equation 39 below).

Disaggregated intermediate input demand

$$QINT_{ca} = ica_{ca} \cdot QINTA_a$$

$$\begin{bmatrix} \text{intermediate demand} \\ \text{for commodity } c \\ \text{from activity } a \end{bmatrix} = f \begin{bmatrix} \text{aggregate intermediate} \\ \text{input quantity} \\ \text{for activity } a \end{bmatrix} \quad \begin{matrix} a \in A \\ c \in C \end{matrix} \quad (17)$$

where

$QINT_{ca}$ quantity of commodity c as intermediate input to activity a

For each activity, the demand for disaggregated intermediate inputs is determined via a standard Leontief formulation as the level of aggregate intermediate input use times a fixed intermediate input coefficient.

Commodity production and allocation

$$QXAC_{ac} + \sum_{h \in H} QHA_{ach} = q_{ac} \cdot QA_a$$

$$\begin{bmatrix} \text{marketed quantity} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} + \begin{bmatrix} \text{household home} \\ \text{consumption} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} = \begin{bmatrix} \text{production} \\ \text{of commodity } c \\ \text{from activity } a \end{bmatrix} \quad \begin{matrix} a \in A \\ c \in CX \end{matrix} \quad (18)$$

where

$QXAC_{ac}$	marketed output quantity of commodity c from activity a
QHA_{ach}	quantity of household home consumption of commodity c from activity a for household h

On the right-hand side, production quantities, disaggregated by activity, are defined as yields times activity levels. On the left-hand side, these quantities are allocated to market sales and home consumption. Note that this equation permits (i) any commodity to be produced by one or more than one activity; and (ii) any activity to produce one or more than one commodity.²⁰

Output Aggregation Function

$$QX_c = a_c^{ac} \cdot \left(\sum_{a \in A} d_{ac}^{ac} \cdot QXAC_{ac}^{-r_c^{ac}} \right)^{-\frac{1}{r_c^{ac}-1}} \quad c \in CX \quad (19)$$

$$\begin{bmatrix} \text{aggregate} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{bmatrix} = CES \begin{bmatrix} \text{activity-specific} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{bmatrix}$$

where

a_c^{ac}	shift parameter for domestic commodity aggregation function
d_{ac}^{ac}	share parameter for domestic commodity aggregation function
r_c^{ac}	domestic commodity aggregation function exponent

First-Order Condition for Output Aggregation Function

$$PXAC_{ac} = PX_c \cdot QX_c \left(\sum_{a \in A} d_{ac}^{ac} \cdot QXAC_{ac}^{-r_c^{ac}} \right)^{-1} \cdot d_{ac}^{ac} \cdot QXAC_{ac}^{-r_c^{ac}-1} \quad \begin{matrix} a \in A \\ c \in CX \end{matrix} \quad (20)$$

$$\begin{bmatrix} \text{marginal cost of com-} \\ \text{modity } c \text{ from activity } a \end{bmatrix} = \begin{bmatrix} \text{marginal revenue product of} \\ \text{commodity } c \text{ from activity } a \end{bmatrix}$$

²⁰ In the SAM, home consumption is represented by payments from households to activities. For the case of home consumption out of activities with multiple outputs, it is necessary to complement the information in the SAM with data on the allocation of consumption across the different activity outputs.

Aggregate marketed production of any commodity is defined as a CES aggregate of the marketed output levels of the different activities producing the commodity (Equation 19). The optimal quantity of the commodity from each activity source is inversely related to the activity-specific price (Equation 20). (QX appears as the output, sold at the price PX and produced with the inputs $QXAC$ that are purchased at the prices $PXAC$.)

More specifically, the choice between commodities from different sources is cast as an optimization problem. Equations 19 and 20 are the first-order conditions for maximizing profits from selling the aggregate output, QX , at the price PX subject to the aggregation function and the disaggregated commodity prices, $PXAC$. A decline in the price $PXAC$ of one activity relative to others would shift demand in its favor without totally eliminating demand for other, higher-price sources. The degree of substitutability between different producers depends on the value of r_c^{ac} (which is a transformation of the elasticity of substitution; cf. footnote 17). Its values (and those of the elasticity) are restricted to assure that the corresponding isoquant is convex to the origin. (In terms of production economics, this is equivalent to a diminishing technical rate of substitution).

It should be noted that, for the case where there is a single producer of a given commodity, the value of the share parameter, d_{ac}^{ac} , would be unity and, as a result, $QXAC = QX$ and $PXAC = PX$, irrespective of the values for the elasticity and the exponent.

Output Transformation (CET) Function

$$QX_c = a_c^t \cdot \left(d_c^t \cdot QE_c^{r_c^t} + (1 - d_c^t) \cdot QD_c^{r_c^t} \right)^{\frac{1}{r_c^t}} \quad c \in (CE \cap CD) \quad (21)$$

$$\left[\begin{array}{c} \text{aggregate marketed} \\ \text{domestic output} \end{array} \right] = CET \left[\begin{array}{c} \text{export quantity, domestic} \\ \text{sales of domestic output} \end{array} \right]$$

where

a_c^t	CET function shift parameter
d_c^t	CET function share parameter
r_c^t	CET function exponent

Equations 21-22 address the allocation of marketed domestic output, defined in Equation 19, to two alternative destinations, domestic sales and exports. Equation 21 reflects the assumption of imperfect transformability between these two destinations. The CET (constant-elasticity-of-transformation) function, which applies to commodities that are both exported and sold domestically, is identical to a CES function except for negative elasticities of substitution. The elasticity of transformation between the two destinations is a transformation of r_c^t , for which the lower limit is one. The values are restricted to assure that the isoquant corresponding to the output transformation function is concave to the origin.²¹

Export-Domestic Supply Ratio

$$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \cdot \frac{1-d_c^t}{d_c^t} \right)^{\frac{1}{r_c^t-1}} \quad c \in (CE \cap CD) \quad (22)$$

$$\left[\begin{matrix} \text{export-domestic} \\ \text{supply ratio} \end{matrix} \right] = f \left[\begin{matrix} \text{export-domestic} \\ \text{price ratio} \end{matrix} \right]$$

Equation 22 defines the optimal mix between exports and domestic sales. Equations 5, 21 and 22 constitute the first-order conditions for maximization of producer revenues given the two prices and subject to the CET function and a fixed quantity of domestic output. Note that Equation 22 assures that an increase in export-domestic price ratio will generate an increase in the export-domestic demand ratio (i.e., a shift toward the destination that offers a higher return).

²¹ For CET functions, $\Omega = \frac{1}{1-r}$, where Ω is the elasticity of transformation and r the exponent. As Ω

varies from zero to infinity, the value of r varies from infinity to one. In Equation 22, as r_c^t approaches one from above, the elasticity of the QE - QD ratio with respect to changes in the PE - PDD ratio gets larger.

Output Transformation for Domestically Sold Outputs without Exports and for Exports without Domestic Sales

$$QX_c = QD_c + QE_c$$

$$\begin{bmatrix} \text{aggregate} \\ \text{marketed} \\ \text{domestic output} \end{bmatrix} = \begin{bmatrix} \text{domestic market} \\ \text{sales of domestic} \\ \text{output [for} \\ c \in (CD \cap CEN)] \end{bmatrix} + \begin{bmatrix} \text{exports [for} \\ c \in (CE \cap CDN)] \end{bmatrix} \quad c \in (CD \cap CEN) \cup (CE \cap CDN) \quad (23)$$

where

$c \in CEN(\subset C)$ non-exported commodities (complement of CE)
 $c \in CDN(\subset C)$ commodities without domestic market sales of domestic output (complement of CD)

This equation replaces the CET function for produced commodities that do not have both exports and domestic sales. It allocates the entire output volume to one of these two destinations.

Composite Supply (Armington) Function

$$QQ_c = a_c^q \cdot \left(d_c^q \cdot QM_c^{-r_c^q} + (1 - d_c^q) \cdot QD_c^{-r_c^q} \right)^{-\frac{1}{r_c^q}} \quad c \in (CM \cap CD) \quad (24)$$

$$\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = f \begin{bmatrix} \text{import quantity, domestic} \\ \text{use of domestic output} \end{bmatrix}$$

where

a_c^q Armington function shift parameter
 d_c^q Armington function share parameter
 r_c^q Armington function exponent

Imperfect substitutability between imports and domestic output sold domestically is captured by a CES (constant elasticity of substitution) aggregation function in which the composite commodity that is supplied domestically is “produced” by domestic and imported commodities, entering this function as “inputs.” This function, with a domain that is limited to commodities that are both imported *and* produced domestically, is often called an “Armington” function, named after the originator of the idea of using a CES function for this purpose. The elasticity of substitution between commodities from these

two sources is a transformation of r_c^q for which the lower limit is minus one (cf. footnote 17).

Import-Domestic Demand Ratio

$$\frac{QM_c}{QD_c} = \left(\frac{PDD_c}{PM_c} \cdot \frac{d_c^q}{1-d_c^q} \right)^{\frac{1}{1+r_c^q}} \quad c \in (CM \cap CD) \quad (25)$$

$$\left[\begin{array}{c} \text{import-domestic} \\ \text{demand ratio} \end{array} \right] = f \left[\begin{array}{c} \text{domestic-import} \\ \text{price ratio} \end{array} \right]$$

Equation 25 defines the optimal mix between imports and domestic output. Its domain is also limited to imports with domestic production. Note that the equation assures that an increase in domestic-import price ratio will generate an increase in the import-domestic demand ratio (i.e., a shift away from the source that became more expensive).²² Together, Equations 4, 24, and 25 constitute the first-order conditions for cost-minimization given the two prices and subject to the Armington function and a fixed quantity of the composite commodity.

Composite Supply for Non-Imported Outputs and Non-Produced Imports

$$QQ_c = QD_c + QM_c$$

$$\left[\begin{array}{c} \text{composite} \\ \text{supply} \end{array} \right] = \left[\begin{array}{c} \text{domestic use of} \\ \text{marketed domestic} \\ \text{output [for} \\ c \in (CD \cap CMN)] \end{array} \right] + \left[\begin{array}{c} \text{imports [for} \\ c \in (CM \cap CDN)] \end{array} \right] \quad c \in [(CD \cap CMN) \cup (CM \cap CDN)] \quad (26)$$

where

$c \in CMN(\subset C)$ set of non-imported commodities

The Armington function is replaced by the above equation for the union of commodities that have either imports or domestic sales of domestic output but not both.

²² See footnote 17 for the definition of the elasticity of substitution. In Equation 25, as the value of r_c^q approaches minus one from above, the elasticity of the import-domestic demand ratio with respect to changes in the $PDD-PM$ ratio gets larger.

For any commodity in this category, it imposes equality between “composite supply” and one of the variables on the right-hand side.

Demand for Transactions Services

$$QT_c = \sum_{c' \in C'} (icm_{cc'} \cdot QM_{c'} + ice_{cc'} \cdot QE_{c'} + icd_{cc'} \cdot QD_{c'})$$

$$\left[\begin{array}{c} \text{demand for} \\ \text{transactions} \\ \text{services} \end{array} \right] = \left[\begin{array}{c} \text{sum of demands} \\ \text{for imports, exports,} \\ \text{and domestic sales} \end{array} \right] \quad c \in CT \quad (27)$$

where

QT_c quantity of commodity demanded as transactions service input

Total demand for trade inputs is the sum of the demands for these inputs that are generated by imports (from moving commodities from the border to domestic demanders), exports (from moving commodities from domestic producers to the border), and domestic market sales (from moving commodities from domestic producers to domestic demanders). In all three cases, fixed quantities of one or more transactions service inputs are required per unit of the traded commodity.

4.3. INSTITUTION BLOCK

Factor income

$$YF_f = \sum_{a \in A} WF_f \cdot \overline{WFDIST}_{fa} \cdot QF_{fa}$$

$$\left[\begin{array}{c} \text{income of} \\ \text{factor } f \end{array} \right] = \left[\begin{array}{c} \text{sum of activity payments} \\ \text{(activity-specific wages} \\ \text{times employment levels)} \end{array} \right] \quad f \in F \quad (28)$$

where

YF_f income of factor f

Institutional factor incomes

$$YIF_{if} = shif_{if} \cdot \left[(1 - tf_f) \cdot YF_f - trnsfr_{row f} \cdot EXR \right]$$

$$\begin{bmatrix} \text{income of} \\ \text{institution } i \\ \text{from factor } f \end{bmatrix} = \begin{bmatrix} \text{share of income} \\ \text{of factor } f \text{ to} \\ \text{institution } i \end{bmatrix} \cdot \begin{bmatrix} \text{income of factor } f \\ \text{(net of tax and} \\ \text{transfer to RoW)} \end{bmatrix} \quad \begin{matrix} i \in INSD \\ f \in F \end{matrix} \quad (29)$$

where

$i \in INS$	set of institutions (domestic and rest of world)
$i \in INSD(\subset INS)$	set of domestic institutions
YIF_{if}	income to domestic institution i from factor f
$shif_{if}$	share for domestic institution i in income of factor f
tf_f	direct tax rate for factor f
$trnsfr_{if}$	transfer from factor f to institution i

Equation 28 defines the total income of each factor. In Equation 29, this income is split among domestic institutions in fixed shares after payment of direct factor taxes and transfers to the rest of the world.²³ The latter are fixed in foreign currency and transformed into domestic currency by multiplying by the exchange rate. In this equation, reference is made to the set of domestic institutions (households, enterprises, and the government), a subset of the set of institutions, which also includes the rest of world.

Income of domestic, non-government institutions

$$YI_i = \sum_{f \in F} YIF_{if} + \sum_{i' \in INSDNG'} TRII_{ii'} + trnsfr_{i gov} \cdot \overline{CPI} + trnsfr_{i row} \cdot EXR$$

$$\begin{bmatrix} \text{income of} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{factor} \\ \text{income} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from other domestic} \\ \text{non-government} \\ \text{institutions} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{government} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{RoW} \end{bmatrix} \quad i \in INSDNG \quad (30)$$

where

$$i \in INSDNG(= INSDNG' \subset INSD)$$

²³ To assure that the total factor income is distributed, it is necessary that $\sum_{i \in INSD} shif_{if} = 1$.

	set of domestic non-government institutions
YI_i	income of institution i (in the set INSDNG)
$TRII_{ii'}$	transfers from institution i' to i (both in the set INSDNG)

Domestic non-government institutions form a subset of the set of domestic institutions. The total income of any domestic non-government institution is the sum of factor incomes (defined in Equation 29), transfers from other domestic non-government institutions (defined below in Equation 31), transfers from the government (indexed to the CPI), and transfers from the rest of the world.²⁴

Intra-Institutional Transfers

$$TRII_{ii'} = shii_{ii'} \cdot (1 - MPS_{i'}) \cdot (1 - TINS_{i'}) \cdot YI_{i'} \quad \begin{matrix} i \in INSDNG \\ i' \in INSDNG' \end{matrix} \quad (31)$$

$$\left[\begin{array}{c} \text{transfer from} \\ \text{institution } i' \text{ to } i \end{array} \right] = \left[\begin{array}{c} \text{share of net income} \\ \text{of institution } i' \\ \text{transferred to } i \end{array} \right] \cdot \left[\begin{array}{c} \text{income of institution} \\ i', \text{ net of savings and} \\ \text{direct taxes} \end{array} \right]$$

where

$shii_{ii'}$	share of net income of i' to i ($i' \in INSDNG'$; $i \in INSDNG$)
MPS_i	marginal propensity to save for domestic non-government institution (exogenous variable)
$TINS_i$	direct tax rate for institution i ($i \in INSDNG$)

Transfers between domestic non-government institutions are paid as fixed shares of the total institutional incomes net of direct taxes and savings. The values of MPS and $TINS$ are defined in separate equations, discussed below.

²⁴ The fact that government transfers are indexed to the *CPI* makes the model homogeneous of degree zero in prices. This indexing influences the results when the *DPI* is the model numéraire. If the *CPI* is the numéraire, it has no effect.

Household Consumption Expenditures

$$EH_h = \left(1 - \sum_{i \in INSDNG} shii_{ih} \right) \cdot (1 - MPS_h) \cdot (1 - TINS_h) \cdot YI_h \quad h \in H \quad (32)$$

$$\begin{bmatrix} \text{household income} \\ \text{disposable for} \\ \text{consumption} \end{bmatrix} = \begin{bmatrix} \text{household income, net of direct} \\ \text{taxes, savings, and transfers to} \\ \text{other non-government institutions} \end{bmatrix}$$

where

$h \in H (\subset INSDNG)$ set of households

EH_h household consumption expenditures

Among the domestic non-government institutions, only households demand commodities. In this equation, the total value of consumption spending is defined as the income that remains after direct taxes, savings, and transfers to other domestic non-government institutions.

Household Consumption Spending on Marketed Commodities

$$PQ_c \cdot QH_{ch} = PQ_c \cdot g_{ch}^m + b_{ch}^m \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot g_{c'h}^m - \sum_{a \in A} \sum_{c' \in C} PXAC_{ac'} \cdot g_{ac'h}^h \right) \quad \begin{matrix} c \in C \\ h \in H \end{matrix} \quad (33)$$

$$\begin{bmatrix} \text{household consumption} \\ \text{spending on market} \\ \text{commodity } c \end{bmatrix} = f \begin{bmatrix} \text{total household consumption} \\ \text{spending, market price of } c, \text{ other} \\ \text{commodity prices (market and home)} \end{bmatrix}$$

where

QH_{ch} quantity of consumption of marketed commodity c for household h

g_{ch}^m subsistence consumption of marketed commodity c for household h

g_{ach}^h subsistence consumption of home commodity c from activity a for household h

b_{ch}^m marginal share of consumption spending on marketed commodity c for household h

Household Consumption Spending on Home Commodities

$$PXAC_{ac} \cdot QHA_{ach} = PXAC_{ac} \cdot g_{ach}^h + b_{ach}^h \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot g_{c'h}^m - \sum_{a \in A} \sum_{c' \in C} PXAC_{ac'} \cdot g_{ac'h}^h \right) \quad \begin{matrix} a \in A \\ c \in C \\ h \in H \end{matrix} \quad (34)$$

$$\left[\begin{matrix} \text{household consumption} \\ \text{spending on home commodity} \\ c \text{ from activity } a \end{matrix} \right] = f \left[\begin{matrix} \text{total household consumption spending,} \\ \text{producer price, other} \\ \text{commodity prices (market and home)} \end{matrix} \right]$$

where

b_{ach}^h marginal share of consumption spending on home commodity c from activity a for household h

It is assumed that each household maximizes a “Stone-Geary” utility function subject to a consumption expenditure constraint. The resulting first-order conditions, Equations 33 and 34, are referred to as LES (Linear Expenditure System) functions since spending on individual commodities is a linear function of total consumption spending, EH . Two functions are needed since household consumption is for two types of commodities: (1) consumption of marketed commodities (purchased at market prices; Equation 33); and (2) consumption of home production (valued at their opportunity cost, the activity-specific producer price, not including marketing costs; Equation 34). Explicit demand functions may be derived by dividing both sides of each equation by the relevant price.

Investment Demand

$$QINV_c = \overline{IADJ} \cdot \overline{qinv}_c$$

$$\left[\begin{matrix} \text{fixed investment} \\ \text{demand for} \\ \text{commodity } c \end{matrix} \right] = \left[\begin{matrix} \text{adjustment factor} \\ \text{times} \\ \text{base-year fixed} \\ \text{investment} \end{matrix} \right] \quad c \in C \quad (35)$$

where

$QINV_c$ quantity of fixed investment demand for commodity
 $\overline{IADJ}$ investment adjustment factor (exogenous variable)
 $\overline{qinv}_c$ base-year quantity of fixed investment demand

Fixed investment demand is defined as the base-year quantity multiplied by an adjustment factor. For the basic model version, the adjustment factor is exogenous, in effect also making the investment quantity exogenous. Inventory investment is also included in the model, but is treated as an exogenous demand (see Equation 40 below).

Government Consumption Demand

$$QG_c = \overline{GADJ} \cdot \overline{qg}_c$$

$$\begin{bmatrix} \text{government} \\ \text{consumption} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = \begin{bmatrix} \text{adjustment factor} \\ \text{times} \\ \text{base-year government} \\ \text{consumption} \end{bmatrix} \quad c \in C \quad (36)$$

where

QG_c	government consumption demand for commodity
$\overline{GADJ}$	government consumption adjustment factor (exogenous variable)
$\overline{qg}_c$	base-year quantity of government demand

Similarly, government consumption demand (in which the main component tends to be the services provided by the government labor force) is also defined as the base-year quantity multiplied by an adjustment factor. This factor is also exogenous and, hence, the quantity of government consumption fixed.

Government Revenue

$$\begin{aligned}
 YG = & \sum_{i \in INSDNG} TINS_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{a \in A} tva_a \cdot PVA_a \cdot QVA_a \\
 & + \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + \sum_{c \in CM} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{c \in CE} te_c \cdot pwe_c \cdot QE_c \cdot EXR \\
 & + \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YF_{gov\ f} + trnsfr_{gov\ row} \cdot EXR
 \end{aligned}$$

$$\left[\begin{array}{c} \text{government} \\ \text{revenue} \end{array} \right] = \left[\begin{array}{c} \text{direct taxes} \\ \text{from} \\ \text{institutions} \end{array} \right] + \left[\begin{array}{c} \text{direct taxes} \\ \text{from} \\ \text{factors} \end{array} \right] + \left[\begin{array}{c} \text{value-} \\ \text{added} \\ \text{tax} \end{array} \right]$$

$$+ \left[\begin{array}{c} \text{activity} \\ \text{tax} \end{array} \right] + \left[\begin{array}{c} \text{import} \\ \text{tariffs} \end{array} \right] + \left[\begin{array}{c} \text{export} \\ \text{taxes} \end{array} \right]$$

$$+ \left[\begin{array}{c} \text{sales} \\ \text{tax} \end{array} \right] + \left[\begin{array}{c} \text{factor} \\ \text{income} \end{array} \right] + \left[\begin{array}{c} \text{transfers} \\ \text{from} \\ \text{RoW} \end{array} \right]$$

where

YG government revenue

Total government revenue is the sum of revenues from taxes, factors, and transfers from the Rest of the World.

Government Expenditures

$$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in INSDNG} trnsfr_{i\ gov} \cdot \overline{CPI}$$

$$\left[\begin{array}{c} \text{government} \\ \text{spending} \end{array} \right] = \left[\begin{array}{c} \text{government} \\ \text{consumption} \end{array} \right] + \left[\begin{array}{c} \text{transfers to domestic} \\ \text{non-government} \\ \text{institutions} \end{array} \right]$$

where

EG government expenditures

Total government spending is the sum of government spending on consumption and transfers.

4.4. SYSTEM CONSTRAINT BLOCK

Factor Markets

$$\sum_{a \in A} QF_{fa} = \overline{QFS}_f \quad f \in F \quad (39)$$

$$\begin{bmatrix} \text{demand for} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{supply of} \\ \text{factor } f \end{bmatrix}$$

where

$\overline{QFS}_f$ quantity supplied of factor (exogenous variable)

This equation imposes equality between the total quantity demanded and total quantity supplied for each factor. In the basic model version, all demand variables are flexible while the supply variable is fixed. The factor wage, WF_f , is the equilibrating variable that assures that this equation is satisfied – an increase in WF_f raises the wage paid by each activity, $W_f \cdot \overline{WFDIST}_{fa}$, which is inversely related to the quantities of factor demand, QF_{fa} . All factors are mobile between the demanding activities.

Two other factor-market closures are programmed in the GAMS version. To specify the case with unemployment at a given wage for a factor, the supply variable for the factor is unfixed (QFS_f) while its economy-wide wage is fixed ($\overline{WF}_f$). The model remains square (one endogenous variable is added but another one removed). Each activity is free to employ the quantity it desires (QF_{fa}) at a fixed wage ($\overline{W}_f \cdot \overline{WFDIST}_{fa}$). The free supply variable, QFS_f , records the total employment level.

Alternatively, to specify the case of a fully segmented factor market with fixed factor demands (for example short-run fixity of non-agricultural capital use), the variables for factor demand and the economy-wide wage are fixed (written $\overline{QF}_{fa}$ and $\overline{WF}_f$) while the variables for supply and wage distortions are unfixed (written QFS_f and $WFDIST_{fa}$). The model remains square (the economy-wide wage variable and a set of activity-specific factor-demand variables are fixed; the supply variable and a set of

activity-specific wage-distortion variables are unfixed). Activity-specific wages, $\overline{WF}_f \cdot WFDIST_{fa}$, vary to assure that the fixed activity-specific employment level, $\overline{QF}_{fa}$, is consistent with profit-maximization (cf. Equation 16). Also for this formulation, the endogenous aggregate factor supply variable merely records the total employment level.

Composite Commodity Markets

$$\begin{aligned}
 QQ_c &= \sum_{a \in A} QINT_{ca} + \sum_{h \in H} QH_{ch} + QG_c \\
 &\quad + QINV_c + qdst_c + QT_c \\
 \begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} &= \begin{bmatrix} \text{intermediate} \\ \text{use} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} \\
 &\quad + \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix} + \begin{bmatrix} \text{trade} \\ \text{input use} \end{bmatrix}
 \end{aligned} \quad c \in C \quad (40)$$

where

$qdst_c$ quantity of stock change

This equation imposes equality between quantities supplied (from Equations 24-26) and demanded of the composite commodity. The demand side includes endogenous terms (from Equations 17, 27, 33, 35, and 36) and a new exogenous term for stock change. Among the endogenous terms, QG and $QINV$ are fixed in the basic model version (cf. Equations 35 and 36). The composite commodity supply, QQ , drives demands for domestic marketed output (QD) and imports (QM). The market-clearing variables are the quantities of import supply, for the import side, and the two interrelated domestic prices, PDD and PDS , for domestic market output.

Current Account Balance for RoW (in Foreign Currency)

$$\begin{aligned}
 \sum_{c \in CM} pwm_c \cdot QM_c + \sum_{f \in F} trnsfr_{row f} &= \sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in INSD} trnsfr_{i row} + \overline{FSAV} \\
 \begin{bmatrix} \text{import} \\ \text{spending} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{transfers} \\ \text{to RoW} \end{bmatrix} &= \begin{bmatrix} \text{export} \\ \text{revenue} \end{bmatrix} + \begin{bmatrix} \text{institutional} \\ \text{transfers} \\ \text{from RoW} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix}
 \end{aligned} \quad (41)$$

where

$\overline{FSAV}$ foreign savings (FCU) (exogenous variable)

The current-account balance (which is expressed in foreign currency) imposes equality between the country's spending and earning of foreign exchange. For the basic model version, foreign savings is fixed; the (real) exchange rate (EXR) serves the role of equilibrating variable to the current-account balance. The fact that all items except imports and exports are fixed means that, in effect, the trade deficit also is fixed. Alternatively, the exchange rate may be fixed and foreign savings unfixed. If so, the trade deficit is free to vary.

Government Balance

$$YG = EG + GSAV$$

$$\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{expenditures} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix} \quad (42)$$

where

$GSAV$ government savings

The government balance imposes equality between current government revenue and the sum of current government expenditures (not including government investment) and savings. The latter may be negative. The alternative mechanisms for assuring that this balance holds are associated with the following equation.

Direct institutional tax rates

$$TINS_i = \overline{tins}_i \cdot \left(1 + \overline{TINSADJ} \cdot tins01_i\right) + \overline{DTINS} \cdot tins01_i$$

$$\begin{bmatrix} \text{direct tax} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix} \quad i \in INSDNG \quad (43)$$

where

$TINS_i$	rate of direct tax on domestic institutions i
$\overline{tins}_i$	exogenous direct tax rate for domestic institution i
$\overline{TINSADJ}$	direct tax scaling factor (= 0 for base; exogenous variable)
$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$\overline{DTINS}$	change in domestic institution tax share (= 0 for base; exogenous variable)

Equation 43 defines the direct tax rates of domestic non-government institutions. For the basic model version, all variables on the right-hand side are fixed, in effect fixing the values for the direct tax rate variable for all institutions; in this setting, government savings is the endogenous variable that clears the government balance.

In the GAMS implementation of the standard model, two alternative closure rules are coded for the government balance (see Section 3.4). For both alternatives, government savings is fixed. In the first case, $DTINS$ is the flexible variable that clears the government balance by scaling the base-year tax rates of each tax-paying institution. In this setting, the rates will change by a uniform number of (percentage) points for all institutions with a value of 1 for the parameter $mps01$. Hence, the initial tax rate has no impact on the rate change. In the second case, $TINSADJ$ is turned into a variable while $DTINS$ is fixed. For this closure, the changes in $TINS$ are relatively large for institutions with relatively large base-year rates. Notice that, when $GSAV$ is fixed for the two alternative closure rules, another variable is made endogenous, thus maintaining a model with an equal number of variables and equations. The choice between alternative closure rules should depend on the empirical context. For example, if the government pursues a policy of raising effective direct tax rates to maintain fixed savings in a setting with a fall in other revenues and/or an increase in government spending, will it raise rates for all or only a subset of the non-government institutions? For the targeted institutions, will the government aim at uniform point increases or will it raise rates in proportion to current rates?

Institutional savings rates

$$MPS_i = \overline{mps}_i \cdot (1 + \overline{MPSADJ} \cdot mps01_i) + DMPS \cdot mps01_i$$

$$\begin{bmatrix} \text{savings} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix} \quad i \in INSDNG \quad (44)$$

where

$\overline{mps}_i$	base savings rate for domestic institution i
$\overline{MPSADJ}$	savings rate scaling factor (= 0 for base)
$mps01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$DMPS$	change in domestic institution savings rates (= 0 for base; exogenous variable)

Equation 44 defines the savings rates of domestic non-government institutions; its structure is the same as that of Equation 43. Whether one or none of the variables $\overline{MPSADJ}$ and $DMPS$ is flexible is related to the closure rule for the savings-investment balance. For the basic model version, $DMPS$ is flexible, permitting MPS to be adjusted by a uniform rate for selected (one or more) non-government institutions.

Savings-Investment Balance

$$\sum_{i \in INSDNG} MPS_i \cdot (1 - TINS_i) \cdot YI_i + GSAV + EXR \cdot \overline{FSAV} =$$

$$\sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$$

$$\begin{bmatrix} \text{non-govern-} \\ \text{ment savings} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix} =$$

$$\begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix} \quad (45)$$

This equation states that total savings and total investment have to be equal. Total savings is the sum of savings from domestic non-government institutions, the

government, and the rest of the world, with the latter item converted into domestic currency. Total investment is the sum of the values of fixed investment (gross fixed capital formation) and stock changes.

In the basic model version, the flexible variable *DMPS* performs the task of clearing this balance (*cf.* Equation 44). None of the other items in the savings-investment balance is free to vary to assure that the balance holds. Given that the balancing role is performed by the savings-side, this closure represents a case of “investment-driven” savings. In the GAMS code, additional savings-investment closures have also been programmed. Under closure 2 (see Table 3), *DMPS* is fixed and *MPSADJ* flexible. For closure 3, for which investment is savings-driven, *IADJ* is flexible whereas both *MPSADJ* and *DMPS* are fixed.

Up to this point, the model that has been stated is not square; the number of equations exceeds the number of variables by one. However, the model satisfies Walras’ law: one equation is functionally dependent on the others and can be dropped. (The savings-investment balance or the current-account balance is commonly eliminated.) After eliminating one equation, the model is square and, in the absence of errors in formulation, a unique solution typically exists. Instead of dropping one equation, it is also possible to add one variable to the macroeconomic balance equations. The solutions value of this variable should be zero – if not, one or more equations are not satisfied and a general equilibrium solution has not been found. This approach is followed in the GAMS version of this model. A variable called *WALRAS* is added to the savings-investment balance. No equation is dropped.

After this adjustment, a complete and self-contained model has been presented. In the basic model version, three more equations (and three new variables that appear in them) are added. The reason for including them is that they permit the formulation of the “balanced” savings-investment closures 4 and 5. After presenting the new equations and their notation, we will return to the closure issue.

Total Absorption

$$\begin{aligned}
 TABS &= \sum_{h \in H} \sum_{c \in C} PQ_c \cdot QH_{ch} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{ac} \cdot QHA_{ach} \\
 &+ \sum_{c \in C} PQ_c \cdot QG_c + \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \\
 \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} &= \begin{bmatrix} \text{household} \\ \text{market} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{home} \\ \text{consumption} \end{bmatrix} \\
 &+ \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}
 \end{aligned} \tag{46}$$

where

$TABS$ total nominal absorption

Total absorption is measured as the total value of domestic final demands (which equals GDP at market prices plus imports minus exports). The new variable, $TABS$, records this value.

Ratio of Investment to Absorption

$$\begin{aligned}
 INVSHR \cdot TABS &= \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c \\
 \begin{bmatrix} \text{investment-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} &= \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}
 \end{aligned} \tag{47}$$

where

$INVSHR$ investment share in nominal absorption

The right-hand side of this equation defines the total investment value (*cf.* Equation 45). On the left-hand side total absorption is multiplied by a new free variable, $INVSHR$. At equilibrium, this variable measures the ratio between investment and absorption.

Ratio of Government Consumption to Absorption

$$GOVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QG_c$$

$$\begin{bmatrix} \text{government} \\ \text{consumption-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} \quad (48)$$

where

GOVSHR government consumption share in nominal absorption

This final equation is similar to Equation 47 except for that investment is replaced by government consumption. The right-hand side defines the value of government consumption (*cf.* Equation 38). On the left-hand side, total absorption is multiplied by a new free variable, *GOVSHR*, which measures the ratio between government consumption and absorption.

The presence of Equations 46-48 and three new variables make it possible to specify savings-investment closures 4 and 5 which represent versions of “bala macroeconomic adjustment that may be preferable for model simulations aimed at generating plausible real-world responses to shocks (*cf.* discussion in Section 3.4). For the investment-driven savings-investment closures 1 and 2, the burden of adjusting to absorption shocks is assumed in full by household consumption.²⁵ Under closure 3, with savings-driven investment, the adjustment burden falls on investment.

Savings-investment closures 4 and 5 in Table 3, which also are programmed in the GAMS version of the model, impose a balanced adjustment in the aggregate components of absorption. Under both, the shares of nominal absorption for investment and government consumption (*INVSHR* and *GOVSHR*) are fixed at base levels while the quantity adjustment factors for fixed investment demand and government consumption

²⁵ However, for simulations with single-period models (like the current model) aimed at exploring welfare impacts of exogenous shocks, savings-investment closure 1 or 2 is often preferable since the model is not able to capture future welfare changes associated with current changes in investment (*cf.* discussion at the end of section 3.4).

(*IADJ* and *GADJ*) are endogenized. The two closures differ in terms of whether *DMPS* or *MPSADJ* is the flexible variable that generates savings-investment equilibrium.

Under these two closures, any change in total absorption would, in nominal terms, be spread evenly across all three components of absorption – given the shares for investment and government consumption, the share for household consumption is implicitly defined. Adjustments in the non-government savings value clear the savings-investment balance. The magnitude of the savings adjustment, which is influenced by changes in investment and government consumption (for the latter via changes in government savings), determines the availability of resources for household consumption.

5. THE STANDARD MODEL IN GAMS

This paper is accompanied by a set of GAMS input files, including country data files that make it possible to conduct simulations with the standard model using data from a selected country. It is straightforward to apply this modeling system to alternative country data sets generated by the user. The purpose of this section is to provide a brief guide to the GAMS files and suggestions about how to use this modeling system. The files themselves include additional explanatory comments.

Table 5 summarizes the contents of the different files that are included and Figure 3 provides a schematic representation for the structure of the GAMS model and data files.²⁶ The modeling system is segmented into two main files, *mod.gms* and *sim.gms*. This segmentation corresponds to two main steps in a typical CGE modeling project. In the first main file, *mod.gms*, the model (which is identical to the model that is presented

²⁶ In addition, the zip file includes an example file for a SAM aggregation program (*samagg.gms*). It may be used independently of the other GAMS files, in which case the appropriately aggregated SAM should be inserted in the country data file. Alternatively, after some adjustments, *samagg.gms* may be used as an include file in the country data set, immediately before the inclusion of *sambal.inc*. If so, the set AC in the country data file should be expanded to include all SAM accounts (both of the initial SAM and the aggregated SAM). The rest of the sets should be defined on the basis of the accounts in the aggregated SAM.

in Section 4 of this paper) is set up and calibrated to a country data set that is read in the form of an include file (<name>.dat). The modeling system includes sample data sets

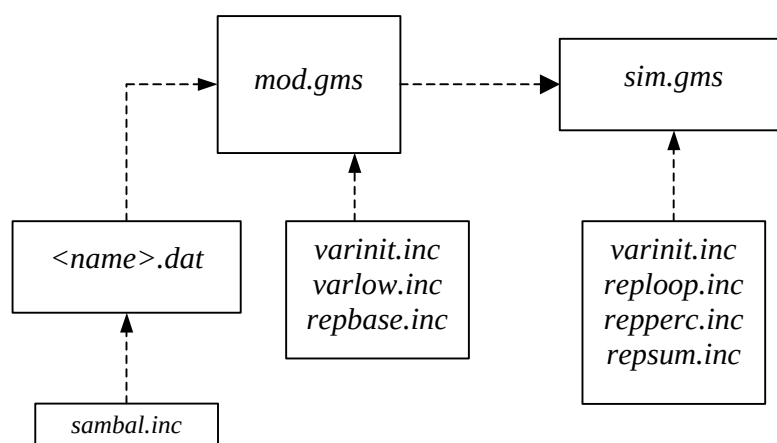
Table 5. File structure in GAMS standard CGE modeling system

<u>File name</u>	<u>Description</u>
<i>mod.gms</i>	All items (sets, parameters, variables) that appear in the standard model equations as well as the equations themselves and the CGE model are declared. Except for the sets, these items are also defined.
<name>.dat	Include files for <i>mod.gms</i> with country-specific data sets (named after the country they represent); one of these files should be included. The data consists of set elements (used to define model sets), a SAM, elasticities, selected physical factor quantities, commodity value shares for home consumption (if needed), and a parameter transforming SAM tax data.
<i>sambal.inc</i>	Include file for <name>.dat. A simple program balances the SAM if its account imbalances exceed a cut-off point.
<i>varinit.inc</i>	Include file for <i>mod.gms</i> (and, optionally, for <i>sim.gms</i>). All model variables are initialized.
<i>varlow.inc</i>	Optional include file for <i>mod.gms</i> . Imposes lower limits on selected model variables.
<i>rebase.inc</i>	Include file for <i>mod.gms</i> . Using data from the base solution, defines an economic structure table, a GDP table, and a Macro SAM.
<i>sim.gms</i>	Restarted from <i>mod.gms</i> . The file includes: a. declarations and definitions of: sets for simulations; experiment parameters; closures for macro system constraints; closures for factor markets; sets for reports; b. declarations of report parameters; c. a loop over the set of current simulations that contains: definitions of simulation-specific parameters and variables; a solve statement; and an include file defining report parameters; d. Additional processing of report parameters (in include files), checks for errors in report parameters, and a display of report parameters.
<i>reploop.inc</i>	Include file for <i>sim.gms</i> . For each simulation, the file defines report parameters for: a. the levels of each model variable;* b. the value of parameters that are subject to change in simulations; c. the incomes and expenditures of each SAM account; d. national accounts data e. macro and factor market closure f. consistency checks for data in c and d.
<i>repper.inc</i>	Include file for <i>sim.gms</i> . For all relevant parameters under a-d in <i>reploop.inc</i> , computation of percentage change from base for non-base simulations (for base simulations.**)
<i>repsum.inc</i>	Include file for <i>sim.gms</i> . Summary results tables based on report parameters defined in <i>reploop.inc</i> and <i>repper.inc</i> .

* These parameters have the same name as the corresponding variable with X added at the end.

** These parameters have the same name as the corresponding parameter in *reploop.inc* with P added at the end.

Figure 3. Structure of GAMS model and data files



that illustrate how data sets should be defined.²⁷ In country data files, the SAMs may be included directly in the GAMS.dat file or be read into this file via a link to a spreadsheet using the program XLLINK (which has to be installed separately). The former approach – direct inclusion of the SAM and the rest of the data – is often preferable since it is less error-prone, and facilitates model documentation and transportability between different users and computers. If the account imbalances in the SAM exceed a low cutoff point, a simple SAM balancing program in the file *sambal.inc* is activated. The file *varinit.inc* is used to initialize all variables at base levels. In the optional file *varlow.inc*, lower limits close to zero are imposed for selected variables as this may improve solver performance.

In *sim.gms*, which restarts from the save files of *mod.gms*, simulations are defined and carried out.²⁸ A note in the beginning of the file specifies the steps that are required

²⁷Three sample data sets are included: (1) test.dat, which is based on data from Mozambique, and is designed to test many of the model features; (2) swazilan.dat, which includes macro data for Swaziland that has only one element in each account set; and (3) zimbabwe.dat, a Zimbabwe data set. When applied to test.dat or swazilan.dat, the standard model can be solved using the student version of GAMS; the Zimbabwe data set is larger and requires a full version of GAMS. Data sets for a number of other countries are also available.

²⁸For save and restart facilities in GAMS, see Brooke et al. (1998), p. 199.

when additional simulations are introduced. For each simulation, the user can choose between alternative closures for macroeconomic constraints (*cf.* Table 3) and factor markets (three alternatives for each factor and simulation; see summary in Section 3.1). The user has the option of selecting the base levels of the model variables as solver starting point selected simulations (by including the file *varinit.inc*); this may facilitate the solver's task of finding a solution relative to the default according to which it uses the variable levels from the preceding model solution. Report parameters are declared in *sim.gms* and defined in include files (*reploop.inc*, *repperc.inc*, and *repsum.inc*). The parameters are designed to contain most of the information that an analyst may be interested in; details are provided in Table 5. The last file may be used as a starting point for user-defined reports that highlight information of interest in any specific application.

This modeling system can be used in a variety of ways, in increasing order of difficulty:

1. The most straightforward is to carry out simulations with one of the existing data sets without any changes in the modeling structure. If so, the user is only required to define new simulations. The file *sim.gms* includes a note that summarizes the core steps when additional simulations are carried out.
2. The user may wish to take the additional step of applying the model to his/her own data set. If so, it is preferable to structure the data set in the same way as in the sample data files. The most critical additional step is to generate a properly formatted SAM. If an available SAM has a different format (for example exports from activity accounts instead of commodity accounts or a different treatment of taxes), we strongly recommend that the user reformat the SAM (a task that can be done inside the GAMS include file). The alternative of adjusting the model code to a differently formatted SAM is likely to be more time-consuming and error-prone. Once the model properly calibrates to the new data set, the user can proceed with simulations.
3. In combination with 1 and 2, more advanced users may also wish to change the model, a step that involves changing the files *mod.gms* and, quite likely,

<name>.dat as existing model elements (sets, parameters, variables and equations) are modified or new ones are declared and defined. If the user also is applying the model to a new data set (point 2 above), it is probably easier to divide the process into two steps, first generating a data set to which the original model calibrates and, as a second step, modify the model. Changes in the model structure will also require the user to modify and/or add to the report system, for example adding new parameters to account for new model variables and modifying the parameters that define the incomes and expenditures of SAM accounts.²⁹

After having read this paper, we recommend that the user familiarize himself/herself with the contents of the different files. For users that limit themselves to the first step, the most important task is to be familiar with the file *sim.gms* and its include files. For users that also add their own database (step 2), it is also crucial to be aware of the detailed structure of the standard SAM (described in Section 2 and exemplified in the country data files) and how it may differ from the original format of any new SAM that the user wants to apply. A thorough study of the modeling system is required for users who, in addition, wish to modify the model, using it as a tool that is further developed in different directions.

²⁹ The modeling system includes consistency checks on the report parameters which will generate error messages if, for example, the reports show imbalances between the income and spending of SAM accounts.

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APPENDIX

Table A.1. Mathematical Summary Statement for the Standard CGE Model

SETS			
<u>Symbol</u>	<u>Explanation</u>	<u>Symbol</u>	<u>Explanation</u>
$a \in A$	activities	$c \in CMN(\subset C)$	commodities not in CM
$a \in ACES(\subset A)$	activities with a CES function at the top of the technology nest	$c \in CT(\subset C)$	transaction service commodities
$a \in ALEO(\subset A)$	activities with a Leontief function at the top of the technology nest	$c \in CX(\subset C)$	commodities with domestic production
$c \in C$	commodities	$f \in F$	factors
$c \in CD(\subset C)$	commodities with domestic sales of domestic output	$i \in INS$	institutions (domestic and rest of world)
$c \in CDN(\subset C)$	commodities not in CD	$i \in INSD(\subset INS)$	domestic institutions
$c \in CE(\subset C)$	exported commodities	$i \in INSDNG(\subset INSD)$	domestic non-government institutions
$c \in CEN(\subset C)$	commodities not in CE	$h \in H(\subset INSDNG)$	households
$c \in CM(\subset C)$	imported commodities		
PARAMETERS			
$cwts_c$	weight of commodity c in the CPI	$\overline{qg}_c$	base-year quantity of government demand
$dwtsc$	weight of commodity c in the producer price index	$\overline{qinv}_c$	base-year quantity of private investment demand
ica_{ca}	quantity of c as intermediate input per unit of activity a	$shif_{if}$	share for domestic institution i in income of factor f
$icd_{cc'}$	quantity of commodity c as trade input per unit of c' produced and sold domestically	$shii_{ii'}$	share of net income of i' to i ($i' \in INSDNG$; $i \in INSDNG$)
$ice_{cc'}$	quantity of commodity c as trade input per exported unit of c'	ta_a	tax rate for activity a
$icm_{cc'}$	quantity of commodity c as trade input per imported unit of c'	te_c	export tax rate
$inta_a$	quantity of aggregate intermediate input per activity unit	tf_f	direct tax rate for factor f
iva_a	quantity of aggregate intermediate input per activity unit	$\overline{tins}_i$	exogenous direct tax rate for domestic institution i
$\overline{mps}_i$	base savings rate for domestic institution i	$tins01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates
$mps01_i$	0-1 parameter with 1 for institutions with potentially flexed direct tax rates	tm_c	import tariff rate
pwe_c	export price (foreign currency)	tq_c	rate of sales tax
pwm_c	import price (foreign currency)	$trnsfr_{if}$	transfer from factor f to institution i
$qdst_c$	quantity of stock change	tva_a	rate of value-added tax for activity a

cont. Table A.1.

Greek Letters

a_a^a	efficiency parameter in the CES activity function	d_c^t	CET function share parameter
a_a^{va}	efficiency parameter in the CES value-added function	d_{fa}^{va}	CES value-added function share parameter for factor f in activity a
a_c^{ac}	shift parameter for domestic commodity aggregation function	g_{ch}^m	subsistence consumption of marketed commodity c for household h
a_c^q	Armington function shift parameter	g_{ach}^h	subsistence consumption of home commodity c from activity a for household h
a_c^t	CET function shift parameter	q_{ac}	yield of output c per unit of activity a
b_{ach}^h	marginal share of consumption spending on home commodity c from activity a for household h	r_a^a	CES production function exponent
b_{ch}^m	marginal share of consumption spending on marketed commodity c for household h	r_a^{va}	CES value-added function exponent
d_a^a	CES activity function share parameter	r_c^{ac}	domestic commodity aggregation function exponent
d_{ac}^{ac}	share parameter for domestic commodity aggregation function	r_c^q	Armington function exponent
d_c^q	Armington function share parameter	r_c^t	CET function exponent

EXOGENOUS VARIABLES

$\overline{CPI}$	consumer price index	$\overline{MPSADJ}$	savings rate scaling factor (= 0 for base)
$\overline{DTINS}$	change in domestic institution tax share (= 0 for base; exogenous variable)	$\overline{QFS}_f$	quantity supplied of factor
$\overline{FSAV}$	foreign savings (FCU)	$\overline{TINSADJ}$	direct tax scaling factor (= 0 for base; exogenous variable)
$\overline{GADJ}$	government consumption adjustment factor	$\overline{WFDIST}_{fa}$	wage distortion factor for factor f in activity a
$\overline{IADJ}$	investment adjustment factor		

cont. Table A.1.

ENDOGENOUS VARIABLES

<i>DMPS</i>	change in domestic institution savings rates (= 0 for base; exogenous variable)	<i>QF_{fa}</i>	quantity demanded of factor f from activity a
<i>DPI</i>	producer price index for domestically marketed output	<i>QG_c</i>	government consumption demand for commodity
<i>EG</i>	government expenditures	<i>QH_{ch}</i>	quantity consumed of commodity c by household h
<i>EH_h</i>	consumption spending for household	<i>QHA_{ach}</i>	quantity of household home consumption of commodity c from activity a for household h
<i>EXR</i>	exchange rate (LCU per unit of FCU)	<i>QINTA_a</i>	quantity of aggregate intermediate input
<i>GOVSHR</i>	government consumption share in nominal absorption	<i>QINT_{ca}</i>	quantity of commodity c as intermediate input to activity a
<i>GSAV</i>	government savings	<i>QINV_c</i>	quantity of investment demand for commodity
<i>INVSHR</i>	investment share in nominal absorption	<i>QM_c</i>	quantity of imports of commodity
<i>MPS_i</i>	marginal propensity to save for domestic non-government institution (exogenous variable)	<i>QQ_c</i>	quantity of goods supplied to domestic market (composite supply)
<i>PA_a</i>	activity price (unit gross revenue)	<i>QT_c</i>	quantity of commodity demanded as trade input
<i>PDD_c</i>	demand price for commodity produced and sold domestically	<i>QVA_a</i>	quantity of (aggregate) value-added
<i>PDS_c</i>	supply price for commodity produced and sold domestically	<i>QX_c</i>	aggregated quantity of domestic output of commodity
<i>PE_c</i>	export price (domestic currency)	<i>QXAC_{ac}</i>	quantity of output of commodity c from activity a
<i>PINTA_a</i>	aggregate intermediate input price for activity a	<i>TABS</i>	total nominal absorption
<i>PM_c</i>	import price (domestic currency)	<i>TINS_i</i>	direct tax rate for institution i (i ∈ INSDNG)
<i>PQ_c</i>	composite commodity price	<i>TRII_{ii'}</i>	transfers from institution i' to i (both in the set INSDNG)
<i>PVA_a</i>	value-added price (factor income per unit of activity)	<i>WF_f</i>	average price of factor
<i>PX_c</i>	aggregate producer price for commodity	<i>YF_f</i>	income of factor f
<i>PXAC_{ac}</i>	producer price of commodity c for activity a	<i>YG</i>	government revenue
<i>QA_a</i>	quantity (level) of activity	<i>YI_i</i>	income of domestic non-government institution
<i>QD_c</i>	quantity sold domestically of domestic output	<i>YIF_{if}</i>	income to domestic institution i from factor f
<i>QE_c</i>	quantity of exports		

cont. Table A.1.

EQUATIONS			
#	Equation	Domain	Description
Price Block			
1	$PM_c = pwm_c \cdot (1 + tm_c) \cdot EXR + \sum_{c' \in CT} PQ_{c'} \cdot icm_{c'c}$ $\begin{bmatrix} \text{import price} \\ \text{(LCU)} \end{bmatrix} = \begin{bmatrix} \text{import price} \\ \text{(FCU)} \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust -} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ \text{(LCU per} \\ \text{FCU)} \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{import unit} \end{bmatrix}$	$c \in CM$	Import Price
2	$PE_c = pwe_c \cdot (1 - te_c) \cdot EXR - \sum_{c' \in CT} PQ_{c'} \cdot ice_{c'c}$ $\begin{bmatrix} \text{export price} \\ \text{(LCU)} \end{bmatrix} = \begin{bmatrix} \text{export price} \\ \text{(FCU)} \end{bmatrix} \cdot \begin{bmatrix} \text{tariff} \\ \text{adjust -} \\ \text{ment} \end{bmatrix} \cdot \begin{bmatrix} \text{exchange rate} \\ \text{(LCU per} \\ \text{FCU)} \end{bmatrix} - \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{export unit} \end{bmatrix}$	$c \in CE$	Export Price
3	$PDD_c = PDS_c + \sum_{c' \in CT} PQ_{c'} \cdot icd_{c'c}$ $\begin{bmatrix} \text{domestic demand} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{domestic supply} \\ \text{price} \end{bmatrix} + \begin{bmatrix} \text{cost of trade} \\ \text{inputs per} \\ \text{unit of} \\ \text{domestic sales} \end{bmatrix}$	$c \in CD$	Demand price of domestic non-traded goods
4	$PQ_c \cdot (1 - tq_c) \cdot QQ_c = PDD_c \cdot QD_c + PM_c \cdot QM_c$ $\begin{bmatrix} \text{absorption} \\ \text{(at demand} \\ \text{prices net of} \\ \text{sales tax)} \end{bmatrix} = \begin{bmatrix} \text{domestic demand price} \\ \text{times} \\ \text{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \text{import price} \\ \text{times} \\ \text{import quantity} \end{bmatrix}$	$c \in (CD \cup CM)$	Absorption
5	$PX_c \cdot QX_c = PDS_c \cdot QD_c + PE_c \cdot QE_c$ $\begin{bmatrix} \text{producer price} \\ \text{times marketed} \\ \text{output quantity} \end{bmatrix} = \begin{bmatrix} \text{domestic supply price} \\ \text{times} \\ \text{domestic sales quantity} \end{bmatrix} + \begin{bmatrix} \text{export price} \\ \text{times} \\ \text{export quantity} \end{bmatrix}$	$c \in CX$	Marketed Output Value
6	$PA_a = \sum_{c \in C} PXAC_{ac} \cdot q_{ac}$ $\begin{bmatrix} \text{activity} \\ \text{price} \end{bmatrix} = \begin{bmatrix} \text{producer prices} \\ \text{times yields} \end{bmatrix}$	$a \in A$	Activity Price

7	$PINTA_a = \sum_{c \in C} PQ_c \cdot ica_{c_a}$ $\begin{bmatrix} \text{aggregate} \\ \text{intermediate} \\ \text{input price} \end{bmatrix} = \begin{bmatrix} \text{intermediate input cost} \\ \text{per unit of aggregate} \\ \text{intermediate input} \end{bmatrix}$	$a \in A$	Aggregate intermediate input price
8	$PA_a \cdot (1 - ta_a) \cdot QA_a = PVA_a \cdot QVA_a + PINTA_a \cdot QINTA_a$ $\begin{bmatrix} \text{activity price} \\ \text{(net of taxes)} \\ \text{times activity level} \end{bmatrix} = \begin{bmatrix} \text{value-added} \\ \text{price times} \\ \text{quantity} \end{bmatrix} + \begin{bmatrix} \text{aggregate} \\ \text{intermediate} \\ \text{input price times} \\ \text{quantity} \end{bmatrix}$	$a \in A$	Activity revenue and costs
9	$\overline{CPI} = \sum_{c \in C} PQ_c \cdot cwtsc_c$ $[CPI] = \begin{bmatrix} \text{prices times} \\ \text{weights} \end{bmatrix}$		Consumer price index
10	$DPI = \sum_{c \in C} PDS_c \cdot dwts_c$ $\begin{bmatrix} \text{Producer price index} \\ \text{for non-traded outputs} \end{bmatrix} = \begin{bmatrix} \text{prices times} \\ \text{weights} \end{bmatrix}$		Producer price index for non-traded market output

Production and commodity block

11	$QA_a = a_a^a \cdot \left(d_a^a \cdot QVA_a^{-r_a^a} + (1 - d_a^a) \cdot QINTA_a^{-r_a^a} \right)^{-\frac{1}{r_a^a}}$ $\begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix} = CES \begin{bmatrix} \text{quantity of aggregate value-added,} \\ \text{quantity aggregate intermediate input} \end{bmatrix}$	$a \in ACES$	CES technology: activity production function
12	$\frac{QVA_a}{QINTA_a} = \left(\frac{PINTA_a}{PVA_a} \cdot \frac{d_a^a}{1 - d_a^a} \right)^{\frac{1}{1 + r_a^a}}$ $\begin{bmatrix} \text{value-added -} \\ \text{intermediate-} \\ \text{input quantity} \\ \text{ratio} \end{bmatrix} = f \begin{bmatrix} \text{intermediate-input} \\ \text{- value-added} \\ \text{price ratio} \end{bmatrix}$	$a \in ACES$	CES technology: Value—Added—Intermediate—Input ratio
13	$QVA_a = iva_a \cdot QA_a$ $\begin{bmatrix} \text{demand for} \\ \text{value-added} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$	$a \in ALEO$	Leontief technology: Demand for aggregate value-added
14	$QINTA_a = inta_a \cdot QA_a$ $\begin{bmatrix} \text{demand for aggregate} \\ \text{intermediate input} \end{bmatrix} = f \begin{bmatrix} \text{activity} \\ \text{level} \end{bmatrix}$	$a \in ALEO$	Leontief technology: Demand for aggregate intermediate input

15	$QVA_a = a_a^{va} \cdot \left(\sum_{f \in F} d_{fa}^{va} \cdot QF_{fa}^{-r_a^{va}} \right)^{-\frac{1}{r_a^{va}}}$ $\left[\begin{array}{c} \text{quantity of aggregate} \\ \text{value-added} \end{array} \right] = CES \left[\begin{array}{c} \text{factor} \\ \text{inputs} \end{array} \right]$	$a \in A$	Value-added and factor demands
16	$W_f \cdot \overline{WFDIST}_{fa} = PVA_a \cdot (1 - tv_a) \cdot QVA_a \cdot \left(\sum_{f \in F'} d_{fa}^{va} \cdot QF_{fa}^{-r_a^{va}} \right)^{-1} \cdot d_{fa}^{va} \cdot QF_{fa}^{-r_a^{va}-1}$ $\left[\begin{array}{c} \text{marginal cost of} \\ \text{factor } f \text{ in activity } a \end{array} \right] = \left[\begin{array}{c} \text{marginal revenue product} \\ \text{of factor } f \text{ in activity } a \end{array} \right]$	$a \in A$ $f \in F$	Factor demand
17	$QINT_{ca} = ica_{ca} \cdot QINTA_a$ $\left[\begin{array}{c} \text{intermediate demand} \\ \text{for commodity } c \\ \text{from activity } a \end{array} \right] = f \left[\begin{array}{c} \text{aggregate intermediate} \\ \text{input quantity} \\ \text{for activity } a \end{array} \right]$	$a \in A$ $c \in C$	Disaggregated intermediate input demand
18	$QXAC_{ac} + \sum_{h \in H} QHA_{ach} = q_{ac} \cdot QA_a$ $\left[\begin{array}{c} \text{marketed quantity} \\ \text{of commodity } c \\ \text{from activity } a \end{array} \right] + \left[\begin{array}{c} \text{household home} \\ \text{consumption} \\ \text{of commodity } c \\ \text{from activity } a \end{array} \right] = \left[\begin{array}{c} \text{production} \\ \text{of commodity } c \\ \text{from activity } a \end{array} \right]$	$a \in A$ $c \in CX$	Commodity production and allocation
19	$QX_c = a_c^{ac} \cdot \left(\sum_{a \in A} d_{ac}^{ac} \cdot QXAC_{ac}^{-r_c^{ac}} \right)^{-\frac{1}{r_c^{ac}-1}}$ $\left[\begin{array}{c} \text{aggregate} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{array} \right] = CES \left[\begin{array}{c} \text{activity-specific} \\ \text{marketed} \\ \text{production of} \\ \text{commodity } c \end{array} \right]$	$c \in CX$	Output Aggregation Function
20	$PXAC_{ac} = PX_c \cdot QX_c \left(\sum_{a \in A'} d_{ac}^{ac} \cdot QXAC_{ac}^{-r_c^{ac}} \right)^{-1} \cdot d_{ac}^{ac} \cdot QXAC_{ac}^{-r_c^{ac}-1}$ $\left[\begin{array}{c} \text{marginal cost of com-} \\ \text{modity } c \text{ from activity } a \end{array} \right] = \left[\begin{array}{c} \text{marginal revenue product of} \\ \text{commodity } c \text{ from activity } a \end{array} \right]$	$a \in A$ $c \in CX$	First-Order Condition for Output Aggregation Function
21	$QX_c = a_c^t \cdot \left(d_c^t \cdot QE_c^{r_c^t} + (1 - d_c^t) \cdot QD_c^{r_c^t} \right)^{\frac{1}{r_c^t}}$ $\left[\begin{array}{c} \text{aggregate marketed} \\ \text{domestic output} \end{array} \right] = CET \left[\begin{array}{c} \text{export quantity, domestic} \\ \text{sales of domestic output} \end{array} \right]$	$c \in (CE \cap CD)$	Output Transformation (CET) Function

22	$\frac{QE_c}{QD_c} = \left(\frac{PE_c}{PDS_c} \cdot \frac{1-d_c^t}{d_c^t} \right)^{\frac{1}{r_c^t-1}}$ $\left[\begin{array}{c} \text{export-domestic} \\ \text{supply ratio} \end{array} \right] = f \left[\begin{array}{c} \text{export-domestic} \\ \text{price ratio} \end{array} \right]$	$c \in (CE \cap CD)$	Export-Domestic Supply Ratio
23	$QX_c = QD_c + QE_c$ $\left[\begin{array}{c} \text{aggregate} \\ \text{marketed} \\ \text{domestic output} \end{array} \right] = \left[\begin{array}{c} \text{domestic market} \\ \text{sales of domestic} \\ \text{output [for} \\ c \in (CD \cap CEN)] \end{array} \right] + \left[\begin{array}{c} \text{exports [for} \\ c \in (CE \cap CDN)] \end{array} \right]$	$c \in$ $(CD \cap CEN)$ $\cup$ $(CE \cap CDN)$	Output Transformation for Non-Exported Commodities
24	$QQ_c = a_c^q \cdot \left(d_c^q \cdot QM_c^{-r_c^q} + (1-d_c^q) \cdot QD_c^{-r_c^q} \right)^{-\frac{1}{r_c^q}}$ $\left[\begin{array}{c} \text{composite} \\ \text{supply} \end{array} \right] = f \left[\begin{array}{c} \text{import quantity, domestic} \\ \text{use of domestic output} \end{array} \right]$	$c \in (CM \cap CD)$	Composite Supply (Armington) Function
25	$\frac{QM_c}{QD_c} = \left(\frac{PDD_c}{PM_c} \cdot \frac{d_c^q}{1-d_c^q} \right)^{\frac{1}{1+r_c^q}}$ $\left[\begin{array}{c} \text{import-domestic} \\ \text{demand ratio} \end{array} \right] = f \left[\begin{array}{c} \text{domestic-import} \\ \text{price ratio} \end{array} \right]$	$c \in (CM \cap CD)$	Import-Domestic Demand Ratio
26	$QQ_c = QD_c + QM_c$ $\left[\begin{array}{c} \text{composite} \\ \text{supply} \end{array} \right] = \left[\begin{array}{c} \text{domestic use of} \\ \text{marketed domestic} \\ \text{output [for} \\ c \in (CD \cap CMN)] \end{array} \right] + \left[\begin{array}{c} \text{imports [for} \\ c \in (CM \cap CDN)] \end{array} \right]$	$c \in$ $(CD \cap CMN)$ $\cup$ $(CM \cap CDN)$	Composite Supply for Non-Imported Outputs and Non-Produced Imports
27	$QT_c = \sum_{c' \in C'} (icm_{cc'} \cdot QM_{c'} + ice_{cc'} \cdot QE_{c'} + icd_{cc'} \cdot QD_{c'})$ $\left[\begin{array}{c} \text{demand for} \\ \text{transactions} \\ \text{services} \end{array} \right] = \left[\begin{array}{c} \text{sum of demands} \\ \text{for imports, exports,} \\ \text{and domestic sales} \end{array} \right]$	$c \in CT$	Demand for Transactions Services

Institution block

28	$YF_f = \sum_{a \in A} WF_f \cdot \overline{WFDIST}_{fa} \cdot QF_{fa}$ $\left[\begin{array}{c} \text{income of} \\ \text{factor } f \end{array} \right] = \left[\begin{array}{c} \text{sum of activity payments} \\ \text{(activity-specific wages} \\ \text{times employment levels)} \end{array} \right]$	$f \in F$	Factor Income
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29	$YIF_{if} = shif_{if} \cdot \left[(1 - tf_f) \cdot YF_f - trnsfr_{row f} \cdot EXR \right]$ $\left[\begin{array}{l} \text{income of} \\ \text{institution } i \\ \text{from factor } f \end{array} \right] = \left[\begin{array}{l} \text{share of income} \\ \text{of factor } f \text{ to} \\ \text{institution } i \end{array} \right] \cdot \left[\begin{array}{l} \text{income of factor } f \\ \text{(net of tax and} \\ \text{transfer to RoW)} \end{array} \right]$	$i \in INSD$ $f \in F$	Institutional factor incomes
30	$YI_i = \sum_{f \in F} YIF_{if} + \sum_{i' \in INSDNG'} TRII_{ii'} + trnsfr_{i gov} \cdot \overline{CPI} + trnsfr_{i row} \cdot EXR$ $\left[\begin{array}{l} \text{income of} \\ \text{institution } i \end{array} \right] = \left[\begin{array}{l} \text{factor} \\ \text{income} \end{array} \right] + \left[\begin{array}{l} \text{transfers} \\ \text{from other domestic} \\ \text{non-government} \\ \text{institutions} \end{array} \right] + \left[\begin{array}{l} \text{transfers} \\ \text{from} \\ \text{government} \end{array} \right] + \left[\begin{array}{l} \text{transfers} \\ \text{from} \\ \text{RoW} \end{array} \right]$	$i \in INSDNG$	Income of domestic, non-government institutions
31	$TRII_{ii'} = shii_{ii'} \cdot (1 - MPS_{i'}) \cdot (1 - TINS_{i'}) \cdot YI_{i'}$ $\left[\begin{array}{l} \text{transfer from} \\ \text{institution } i' \text{ to } i \end{array} \right] = \left[\begin{array}{l} \text{share of net income} \\ \text{of institution } i' \\ \text{transferred to } i \end{array} \right] \cdot \left[\begin{array}{l} \text{income of institution} \\ i', \text{ net of savings and} \\ \text{direct taxes} \end{array} \right]$	$i \in INSDNG$ $i' \in INSDNG'$	Intra-Institutional Transfers
32	$EH_h = \left(1 - \sum_{i \in INSDNG} shii_{ih} \right) \cdot (1 - MPS_h) \cdot (1 - TINS_h) \cdot YI_h$ $\left[\begin{array}{l} \text{household income} \\ \text{disposable for} \\ \text{consumption} \end{array} \right] = \left[\begin{array}{l} \text{household income, net of direct} \\ \text{taxes, savings, and transfers to} \\ \text{other non-government institutions} \end{array} \right]$	$h \in H$	Household Consumption Expenditure
33	$QH_{ch} = g_{ch} + \frac{b_{ch}^m \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot g_{c'h}^m - \sum_{a \in A} \sum_{c' \in C} PXAC_{ac'} \cdot g_{ac'h}^h \right)}{PQ_c}$ $\left[\begin{array}{l} \text{quantity of} \\ \text{household demand} \\ \text{for commodity } c \end{array} \right] = f \left[\begin{array}{l} \text{household} \\ \text{consumption} \\ \text{spending,} \\ \text{market price} \end{array} \right]$	$c \in C$ $h \in H$	Household Consumption Demand for Marketed commodities
34	$QHA_{ach} = g_{ach}^h + \frac{b_{ach}^h \cdot \left(EH_h - \sum_{c' \in C} PQ_{c'} \cdot g_{c'h}^m - \sum_{a \in A} \sum_{c' \in C} PXAC_{ac'} \cdot g_{ac'h}^h \right)}{PXAC_{ac}}$ $\left[\begin{array}{l} \text{quantity of} \\ \text{household demand} \\ \text{for home commodity } c \\ \text{from activity } a \end{array} \right] = f \left[\begin{array}{l} \text{household} \\ \text{disposable} \\ \text{income,} \\ \text{producer price} \end{array} \right]$	$a \in A$ $c \in C$ $h \in H$	Household Consumption Demand for Home Commodities
35	$QINV_c = \overline{IADJ} \cdot \overline{qinv}_c$ $\left[\begin{array}{l} \text{fixed investment} \\ \text{demand for} \\ \text{commodity } c \end{array} \right] = \left[\begin{array}{l} \text{adjustment factor} \\ \text{times} \\ \text{base-year fixed} \\ \text{investment} \end{array} \right]$	$c \in CINV$	Investment Demand

36	$QG_c = \overline{GADJ} \cdot \overline{qg}_c$ $\begin{bmatrix} \text{government} \\ \text{consumption} \\ \text{demand for} \\ \text{commodity } c \end{bmatrix} = \begin{bmatrix} \text{adjustment factor} \\ \text{times} \\ \text{base-year government} \\ \text{consumption} \end{bmatrix}$	$c \in C$	Government Consumption Demand
37	$YG = \sum_{i \in INSDNG} TINS_i \cdot YI_i + \sum_{f \in F} tf_f \cdot YF_f + \sum_{a \in A} tva_a \cdot PVA_a \cdot QVA_a$ $+ \sum_{a \in A} ta_a \cdot PA_a \cdot QA_a + \sum_{c \in CM} tm_c \cdot pwm_c \cdot QM_c \cdot EXR + \sum_{c \in CE} te_c \cdot pwe_c \cdot QE_c \cdot EXR$ $+ \sum_{c \in C} tq_c \cdot PQ_c \cdot QQ_c + \sum_{f \in F} YF_{gov f} + trnsfr_{gov row} \cdot EXR$ $\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{direct taxes} \\ \text{from} \\ \text{institutions} \end{bmatrix} + \begin{bmatrix} \text{direct taxes} \\ \text{from} \\ \text{factors} \end{bmatrix} + \begin{bmatrix} \text{value-} \\ \text{added} \\ \text{tax} \end{bmatrix}$ $+ \begin{bmatrix} \text{activity} \\ \text{tax} \end{bmatrix} + \begin{bmatrix} \text{import} \\ \text{tariffs} \end{bmatrix} + \begin{bmatrix} \text{export} \\ \text{taxes} \end{bmatrix}$ $+ \begin{bmatrix} \text{sales} \\ \text{tax} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{income} \end{bmatrix} + \begin{bmatrix} \text{transfers} \\ \text{from} \\ \text{RoW} \end{bmatrix}$		Government Revenue
38	$EG = \sum_{c \in C} PQ_c \cdot QG_c + \sum_{i \in INSDNG} trnsfr_{i gov} \cdot \overline{CPI}$ $\begin{bmatrix} \text{government} \\ \text{spending} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{transfers to domestic} \\ \text{non-government} \\ \text{institutions} \end{bmatrix}$		Government Expenditures

System Constraint Block

39	$\sum_{a \in A} QF_{f a} = \overline{QFS}_f$ $\begin{bmatrix} \text{demand for} \\ \text{factor } f \end{bmatrix} = \begin{bmatrix} \text{supply of} \\ \text{factor } f \end{bmatrix}$	$f \in F$	Factor market
40	$QQ_c = \sum_{a \in A} QINT_{c a} + \sum_{h \in H} QH_{c h} + QG_c$ $+ QINV_c + qdst_c + QT_c$ $\begin{bmatrix} \text{composite} \\ \text{supply} \end{bmatrix} = \begin{bmatrix} \text{intermediate} \\ \text{use} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix}$ $+ \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix} + \begin{bmatrix} \text{trade} \\ \text{input use} \end{bmatrix}$	$c \in C$	Composite Commodity Markets
41	$\sum_{c \in CM} pwm_c \cdot QM_c + \sum_{f \in F} trnsfr_{row f} = \sum_{c \in CE} pwe_c \cdot QE_c + \sum_{i \in INSD} trnsfr_{i row} + \overline{FSAV}$ $\begin{bmatrix} \text{import} \\ \text{spending} \end{bmatrix} + \begin{bmatrix} \text{factor} \\ \text{transfers} \\ \text{to RoW} \end{bmatrix} = \begin{bmatrix} \text{export} \\ \text{revenue} \end{bmatrix} + \begin{bmatrix} \text{institutional} \\ \text{transfers} \\ \text{from RoW} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix}$		Current Account Balance for RoW (in Foreign Currency)

42	$YG = EG + GSAV$ $\begin{bmatrix} \text{government} \\ \text{revenue} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{expenditures} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix}$		Government Balance
43	$TINS_i = \overline{tins}_i \cdot (1 + \overline{TINSADJ} \cdot \overline{tins01}_i) + \overline{DTINS} \cdot \overline{tins01}_i$ $\begin{bmatrix} \text{direct tax} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix}$	$i \in INSDNG$	Direct institutional tax rates
44	$MPS_i = \overline{mps}_i \cdot (1 + \overline{MPSADJ} \cdot \overline{mps01}_i) + \overline{DMPS} \cdot \overline{mps01}_i$ $\begin{bmatrix} \text{savings} \\ \text{rate for} \\ \text{institution } i \end{bmatrix} = \begin{bmatrix} \text{base rate adjusted} \\ \text{for scaling for} \\ \text{selected institutions} \end{bmatrix} + \begin{bmatrix} \text{point change} \\ \text{for selected} \\ \text{institutions} \end{bmatrix}$	$i \in INSDNG$	Institutional savings rates
45	$\sum_{i \in INSDNG} MPS_i \cdot (1 - TINS_i) \cdot YI_i + GSAV + EXR \cdot \overline{FSAV} =$ $\sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$ $\begin{bmatrix} \text{non-govern-} \\ \text{ment savings} \end{bmatrix} + \begin{bmatrix} \text{government} \\ \text{savings} \end{bmatrix} + \begin{bmatrix} \text{foreign} \\ \text{savings} \end{bmatrix} =$ $\begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}$		Savings-Investment Balance
46	$TABS = \sum_{h \in H} \sum_{c \in C} PQ_c \cdot QH_{ch} + \sum_{a \in A} \sum_{c \in C} \sum_{h \in H} PXAC_{ac} \cdot QHA_{ach}$ $+ \sum_{c \in C} PQ_c \cdot QG_c + \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$ $\begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{household} \\ \text{market} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{household} \\ \text{home} \\ \text{consumption} \end{bmatrix}$ $+ \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}$		Total Absorption
47	$INVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QINV_c + \sum_{c \in C} PQ_c \cdot qdst_c$ $\begin{bmatrix} \text{investment-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{fixed} \\ \text{investment} \end{bmatrix} + \begin{bmatrix} \text{stock} \\ \text{change} \end{bmatrix}$		Ratio of Investment to Absorption
48	$GOVSHR \cdot TABS = \sum_{c \in C} PQ_c \cdot QG_c$ $\begin{bmatrix} \text{government} \\ \text{consumption-} \\ \text{absorption} \\ \text{ratio} \end{bmatrix} \cdot \begin{bmatrix} \text{total} \\ \text{absorption} \end{bmatrix} = \begin{bmatrix} \text{government} \\ \text{consumption} \end{bmatrix}$		Ratio of Government Consumption to Absorption

Appendix 2. Core GAMS Code for Standard CGE Model

SETS

AC	global set for model accounts-aggregated microsam accounts
A(AC)	activities
ACES(A)	activities with CES fn at top of technology nest
ALEO(A)	activities with Leontief fn at top of technology nest
C(AC)	commodities
CD(C)	commodities with domestic sales of output
CDN(C)	commodities without domestic sales of output
CE(C)	exported commodities
CEN(C)	non-export commodities
CM(C)	imported commodities
CMN(C)	non-imported commodities
CX(C)	commodities with output
F(AC)	factors
INS(AC)	institutions
INSD(INS)	domestic institutions
INSDNG(INSD)	domestic non-government institutions
H(INSDNG)	households

PARAMETERS

*Parameters other than tax rates

alphaa(A)	shift parameter for top level CES function
alphaac(C)	shift parameter for domestic commodity aggregation fn
alphaq(C)	shift parameter for Armington function
alphat(C)	shift parameter for CET function
alphava(A)	shift parameter for CES activity production function
betah(A,C,H)	marg shr of hhd cons on home com c from act a
betam(C,H)	marg share of hhd cons on marketed commodity c
cwts(C)	consumer price index weights
deltaa(A)	share parameter for top level CES function
deltaac(A,C)	share parameter for domestic commodity aggregation fn
deltaq(C)	share parameter for Armington function
deltat(C)	share parameter for CET function
deltava(F,A)	share parameter for CES activity production function
dwts(C)	domestic sales price weights
gammah(A,C,H)	per-cap subsist cons for hhd h on home com c fr act a
gammam(C,H)	per-cap subsist cons of market com c for hhd h
ica(C,A)	intermediate input c per unit of aggregate intermediate
inta(A)	aggregate intermediate input coefficient
iva(A)	aggregate value added coefficient
icd(C,CP)	trade input of c per unit of comm'y cp produced & sold
dom'ly	
ice(C,CP)	trade input of c per unit of comm'y cp exported
icm(C,CP)	trade input of c per unit of comm'y cp imported
mps01(INS)	0-1 par for potential flexing of savings rates
mpsbar(INS)	marg prop to save for dom non-gov inst ins (exog part)

qdst(C)	inventory investment by sector of origin
qbarg(C)	exogenous (unscaled) government demand
qbarinv(C)	exogenous (unscaled) investment demand
rhoa(A)	CES top level function exponent
rhoac(C)	domestic commodity aggregation function exponent
rhoq(C)	Armington function exponent
rhot(C)	CET function exponent
rhova(A)	CES activity production function exponent
shif(INS,F)	share of dom. inst'on i in income of factor f
shii(INS,INSP)	share of inst'on i in post-tax post-sav income of inst ip
supernum(H)	LES supernumerary income
theta(A,C)	yield of commodity c per unit of activity a
tins01(INS)	0-1 par for potential flexing of dir tax rates
trnsfr(INS,AC)	transfers fr inst. or factor ac to institution ins

*Tax rates

ta(A)	rate of tax on producer gross output value
te(C)	rate of tax on exports
tf(F)	rate of direct tax on factors (soc sec tax)
tinsbar(INS)	rate of (exog part of) direct tax on dom inst ins
tm(C)	rate of import tariff
tq(C)	rate of sales tax
tva(A)	rate of value-added tax

VARIABLES

CPI	consumer price index (PQ-based)
DPI	index for domestic producer prices (PDS-based)
DMPS	change in marginal propensity to save for selected inst
DTINS	change in domestic institution tax share
EG	total current government expenditure
EH(H)	household consumption expenditure
EXR	exchange rate
FSAV	foreign savings
GADJ	government demand scaling factor
GOVSHR	govt consumption share of absorption
GSAV	government savings
IADJ	investment scaling factor (for fixed capital formation)
INVSHR	investment share of absorption
MPS(INS)	marginal propensity to save for dom non-gov inst ins
MPSADJ	savings rate scaling factor
PA(A)	output price of activity a
PDD(C)	demand price for com'y c produced & sold domestically
PDS(C)	supply price for com'y c produced & sold domestically
PE(C)	price of exports
PINTA(A)	price of intermediate aggregate
PM(C)	price of imports
PQ(C)	price of composite good c
PVA(A)	value added price
PWE(C)	world price of exports
PWM(C)	world price of imports
PX(C)	average output price
PXAC(A,C)	price of commodity c from activity a
QA(A)	level of domestic activity
QD(C)	quantity of domestic sales

QE(C) quantity of exports
 QF(F,A) quantity demanded of factor f from activity a
 QFS(F) quantity of factor supply
 QG(C) quantity of government consumption
 QH(C,H) quantity consumed of marketed commodity c by household h
 QHA(A,C,H) quantity consumed of home commodity c fr act a by hhd h
 QINT(C,A) quantity of intermediate demand for c from activity a
 QINTA(A) quantity of aggregate intermediate input
 QINV(C) quantity of fixed investment demand
 QM(C) quantity of imports
 QQ(C) quantity of composite goods supply
 QT(C) quantity of trade and transport demand for commodity c
 QVA(A) quantity of aggregate value added
 QX(C) quantity of aggregate marketed commodity output
 QXAC(A,C) quantity of ouput of commodity c from activity a
 TABS total absorption
 TINS(INS) rate of direct tax on domestic institutions ins
 TINSADJ direct tax scaling factor
 TRII(INS,INSP) transfers to dom. inst. insdng from insdngp
 WALRAS savings-investment imbalance (should be zero)
 WF(F) economy-wide wage (rent) for factor f
 WFDIST(F,A) factor wage distortion variable
 YF(F) factor income
 YG total current government income
 YIF(INS,F) income of institution ins from factor f
 YI(INS) income of (domestic non-governmental) institution ins
 ;

EQUATIONS

*Price block=====

PMDEF(C) domestic import price
 PEDEF(C) domestic export price
 PDDDEF(C) dem price for com'y c produced and sold domestically
 PQDEF(C) value of sales in domestic market
 PXDEF(C) value of marketed domestic output
 PADEF(A) output price for activity a
 PINTADEF(A) price of aggregate intermediate input
 PVADEF(A) value-added price
 CPIDEF consumer price index
 DPIDEF domestic producer price index

*Production and trade block=====

CESAGGPRD(A) CES aggregate prod fn (if CES top nest)
 CESAGGFOC(A) CES aggregate first-order condition (if CES top nest)
 LEOAGGINT(A) Leontief aggreg intermed dem (if Leontief top nest)
 LEOAGGVA(A) Leontief aggreg value-added dem (if Leontief top nest)
 CESVAPRD(A) CES value-added production function
 CESVAFOC(F,A) CES value-added first-order condition
 INTDEM(C,A) intermediate demand for commodity c from activity a
 COMPRDFN(A,C) production function for commodity c and activity a
 OUTAGGFN(C) output aggregation function
 OUTAGGFOC(A,C) first-order condition for output aggregation function
 CET(C) CET function

CET2(C) domestic sales and exports for outputs without both
 ESUPPLY(C) export supply
 ARMINGTON(C) composite commodity aggregation function
 COSTMIN(C) first-order condition for composite commodity cost min
 ARMINGTON2(C) comp supply for com's without both dom. sales and imports
 QTDEM(C) demand for transactions (trade and transport) services

*Institution block =====

YFDEF(F) factor incomes
 YIFDEF(INS,F) factor incomes to domestic institutions
 YIDEF(INS) total incomes of domest non-gov't institutions
 EHDEF(H) household consumption expenditures
 TRIIDEF(INS,INSP) transfers to inst'on ins from inst'on insp
 HMDEM(C,H) LES cons demand by hhd h for marketed commodity c
 HADEM(A,C,H) LES cons demand by hhd h for home commodity c fr act a
 INVDEM(C) fixed investment demand
 GOVDEM(C) government consumption demand
 EGDEF total government expenditures
 YGDEF total government income

*System constraint block=====

COMEQUIL(C) composite commodity market equilibrium
 FACEQUIL(F) factor market equilibrium
 CURACCBAL current account balance (of RoW)
 GOVBAL government balance
 TINSDEF(INS) direct tax rate for inst ins
 MPSDEF(INS) marg prop to save for inst ins
 SAVINVBAL savings-investment balance
 TABSEQ total absorption
 INVABEQ investment share in absorption
 GDABEQ government consumption share in absorption
 ;

*Notational convention inside equations:

*Parameters and "invariably" fixed variables are in lower case.
 *Potentially "variable" variables are in upper case.

*Price block=====

PMDEF(C)\$CM(C)..
 $PM(C) = E = p_{wm}(C) * (1 + t_m(C)) * EXR + \sum(CT, PQ(CT) * icm(CT, C));$
 PEDEF(C)\$CE(C)..
 $PE(C) = E = p_{we}(C) * (1 - t_e(C)) * EXR - \sum(CT, PQ(CT) * ice(CT, C));$
 PDDDEF(C)\$CD(C).. PDD(C) =E= PDS(C) + $\sum(CT, PQ(CT) * icd(CT, C));$
 PQDEF(C)\$CD(C) OR CM(C)..
 $PQ(C) * (1 - t_q(C)) * QQ(C) = E = PDD(C) * QD(C) + PM(C) * QM(C);$
 PXDEF(C)\$CX(C).. $PX(C) * QX(C) = E = PDS(C) * QD(C) + PE(C) * QE(C);$
 PADEF(A).. $PA(A) = E = \sum(C, PXAC(A, C) * \theta(A, C));$

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PINTADEF(A).. PINTA(A) =E= SUM(C, PQ(C)*ica(C,A)) ;

PVADEF(A).. PA(A)*(1-ta(A))*QA(A) =E= PVA(A)*QVA(A) + PINTA(A)*QINTA(A) ;

CPIDEF.. CPI =E= SUM(C, cwts(C)*PQ(C)) ;

DPIDEF.. DPI =E= SUM(CD, dwts(CD)*PDS(CD)) ;

*Production and trade block=====

*CESAGGPRD and CESAGGFOC apply to activities with CES function at
*top of technology nest.

CESAGGPRD(A)$ACES(A)..
  QA(A) =E= alphas(A)*(deltaa(A)*QVA(A)**(-rhoa(A))
    + (1-deltaa(A))*QINTA(A)**(-rhoa(A))**(-1/rhoa(A)) ;

CESAGGFOC(A)$ACES(A)..
  QVA(A) =E= QINTA(A)*((PINTA(A)/PVA(A))*(deltaa(A)/
    (1 - deltaa(A))))**1/(1+rhoa(A)) ;

*LEOAGGINT and LEOAGGVA apply to activities with Leontief function at
*top of technology nest.

LEOAGGINT(A)$ALEO(A).. QINTA(A) =E= inta(A)*QA(A) ;

LEOAGGVA(A)$ALEO(A).. QVA(A) =E= iva(A)*QA(A) ;

*CESVAPRD, CESVAFOC, INTDEM apply at the bottom of the technology nest
*(for all activities).

CESVAPRD(A)..
  QVA(A) =E= alphava(A)*(SUM(F,
    deltava(F,A)*QF(F,A)**(-rhova(A))) )**(-1/rhova(A)) ;

CESVAFOC(F,A)$deltava(F,A)..
  WF(F)*wfdist(F,A) =E=
  PVA(A)*(1-tva(A))
  * QVA(A) * SUM(FP, deltava(FP,A)*QF(FP,A)**(-rhova(A)) )**(-1)
  *deltava(F,A)*QF(F,A)**(-rhova(A)-1);

INTDEM(C,A)$ica(C,A).. QINT(C,A) =E= ica(C,A)*QINTA(A);

COMPRDFN(A,C)$theta(A,C)..
  QXAC(A,C) + SUM(H, QHA(A,C,H)) =E= theta(A,C)*QA(A) ;

OUTAGGFN(C)$CX(C)..
  QX(C) =E= alphaac(C)*SUM(A, deltaac(A,C)*QXAC(A,C)
    **(-rhoac(C))**(-1/rhoac(C)));

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OUTAGGFOC(A,C)$deltaac(A,C)..
  PXAC(A,C) =E=
  PX(C)
  * QX(C) * SUM(AP, deltaac(AP,C)*QXAC(AP,C)**(-rhoac(C)) )**(-1)
  *deltaac(A,C)*QXAC(A,C)**(-rhoac(C)-1);

CET(C)$CE(C) AND CD(C)..
  QX(C) =E= alphas(C)*(deltat(C)*QE(C)**rhot(C) +
    (1 - deltat(C))*QD(C)**rhot(C))**1/rhot(C)) ;

ESUPPLY(C)$CE(C) AND CD(C)..
  QE(C) =E= QD(C)*((PE(C)/PDS(C))*
    ((1 - deltat(C))/deltat(C)))**1/(rhot(C)-1)) ;

CET2(C)$ ( CD(C) AND CEN(C)) OR (CE(C) AND CDN(C)) ..
  QX(C) =E= QD(C) + QE(C);

ARMINGTON(C)$CM(C) AND CD(C)..
  QQ(C) =E= alphas(C)*(deltaq(C)*QM(C)**(-rhoq(C)) +
    (1 - deltaq(C))*QD(C)**(-rhoq(C))**(-1/rhoq(C)) ;

COSTMIN(C)$CM(C) AND CD(C)..
  QM(C) =E= QD(C)*((PDD(C)/PM(C))*(deltaq(C)/(1 - deltaq(C)))
    **1/(1 + rhoq(C)));

ARMINGTON2(C)$ ( CD(C) AND CMN(C)) OR (CM(C) AND CDN(C)) ..
  QQ(C) =E= QD(C) + QM(C);

QTDEN(C)$CT(C)..
  QT(C) =E= SUM(CP, icm(C,CP)*QM(CP) + ice(C,CP)*QE(CP) + icd(C,CP)*QD(CP));

*Institution block =====

YFDEF(F).. YF(F) =E= SUM(A, WF(F)*wfdist(F,A)*QF(F,A));

YIFDEF(INSDF)$shif(INSDF)..
  YIF(INSDF) =E= shif(INSDF)*((1-tf(f))*YF(F) - trnsfr('ROW',F)*EXR);

YIDEF(INSNG)..
  YI(INSNG) =E=
  SUM(F, YIF(INSNG,F)) + SUM(INSNGP, TRII(INSNG,INSNGP))
  + trnsfr(INSNG,'GOV')*CPI + trnsfr(INSNG,'ROW')*EXR;

TRIIDEF(INSNG,INSNGP)$shii(INSNG,INSNGP)..
  TRII(INSNG,INSNGP) =E= shii(INSNG,INSNGP)
    * (1 - MPS(INSNGP)) * (1 - TINS(INSNGP)) * YI(INSNGP);

EHDEF(H)..
  EH(H) =E= (1 - SUM(INSNG, shii(INSNG,H))) * (1 - MPS(H))
    * (1 - TINS(H)) * YI(H);

HMDEM(C,H)$betam(C,H)..
  PQ(C)*QH(C,H) =E=

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PQ(C)*gammam(C,H)
+ betam(C,H)*( EH(H) - SUM(CP, PQ(CP)*gammam(CP,H))
- SUM((A,CP), PXAC(A,CP)*gammah(A,CP,H))) );

HADEM(A,C,H)$betah(A,C,H)..
PXAC(A,C)*QHA(A,C,H) =E=
PXAC(A,C)*gammah(A,C,H)
+ betah(A,C,H)*(EH(H) - SUM(CP, PQ(CP)*gammam(CP,H))
- SUM((AP,CP), PXAC(AP,CP)*gammah(AP,CP,H))) );

INVDEM(C).. QINV(C) =E= IADJ*qbarinv(C);

GOVDEM(C).. QG(C) =E= GADJ*qbarg(C);

YGDEF..
YG =E= SUM(INSNG, TINS(INSNG)*YI(INSNG))
+ SUM(F, tf(F)*YF(F))
+ SUM(A, tva(A)*PVA(A)*QVA(A))
+ SUM(A, ta(A)*PA(A)*QA(A))
+ SUM(C, tm(C)*pwm(C)*QM(C))*EXR
+ SUM(C, te(C)*pwe(C)*QE(C))*EXR
+ SUM(C, tq(C)*PQ(C)*QQ(C))
+ SUM(F, YIF('GOV',F))
+ trnsfr('GOV','ROW')*EXR;

EGDEF.. EG =E= SUM(C, PQ(C)*QG(C)) + SUM(INSNG, trnsfr(INSNG,'GOV'))*CPI;

*System constraint block=====

FACEQUIL(F).. SUM(A, QF(F,A)) =E= QFS(F);

COMEQUIL(C)..
QQ(C) =E= SUM(A, QINT(C,A)) + SUM(H, QH(C,H)) + QG(C)
+ QINV(C) + qdst(C) + QT(C);

CURACCBAL..
SUM(C, pwm(C)*QM(C)) + SUM(F, trnsfr('ROW',F)) =E=
SUM(C, pwe(C)*QE(C)) + SUM(INSNG, trnsfr(INSNG,'ROW')) + FSAV;

GOVBAL.. YG =E= EG + GSAV;

TINSDEF(INSNG)..
TINS(INSNG) =E= tinsbar(INSNG)*(1 + TINSADJ*tins01(INSNG)) +
DTINS*tins01(INSNG);

MPSDEF(INSNG)..
MPS(INSNG) =E= mpsbar(INSNG)*(1 + MPSADJ*mps01(INSNG)) +
DMPS*mps01(INSNG);

SAVINVBAL..
SUM(INSNG, MPS(INSNG) * (1 - TINS(INSNG)) * YI(INSNG))
+ GSAV + FSAV*EXR =E=
SUM(C, PQ(C)*QINV(C)) + SUM(C, PQ(C)*qdst(C)) + WALRAS;

TABSEQ..
TABS =E=
SUM((C,H), PQ(C)*QH(C,H)) + SUM((A,C,H), PXAC(A,C)*QHA(A,C,H))
+ SUM(C, PQ(C)*QG(C)) + SUM(C, PQ(C)*QINV(C)) + SUM(C, PQ(C)*qdst(C));

INVABEQ.. INVSHR*TABS =E= SUM(C, PQ(C)*QINV(C)) + SUM(C, PQ(C)*qdst(C));

GDABEQ.. GOVSHR*TABS =E= SUM(C, PQ(C)*QG(C));

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